

Équations

Exercice 1.

a) $-18x + 9 - 20x - 12 = -8x - 8$

$$\Leftrightarrow -30x = -5 \quad \Leftrightarrow x = \frac{1}{6}$$

$$\mathcal{S} = \left\{ \frac{1}{6} \right\}$$

b) $20x + 36 - 4x - 20 = 7x - 2$

$$\Leftrightarrow 9x = -18 \quad \Leftrightarrow x = -2$$

$$\mathcal{S} = \{-2\}$$

c) $x^2 - 6x + 8 + x^2 - 4x + 3 = 39$

$$\Leftrightarrow 2x^2 - 10x - 28 = 0$$

$$\Leftrightarrow 2(x^2 - 5x - 14) = 0$$

$$\Leftrightarrow 2(x - 7)(x + 2) = 0$$

$$\mathcal{S} = \{-2; 7\}$$

d) $(3x + 4)(2x + 1) = 0$

$$\mathcal{S} = \left\{ -\frac{4}{3}; -\frac{1}{2} \right\}$$

$40x + 40 - 54x - 63 = 36x - 24$

$$\Leftrightarrow -50x = -1 \quad \Leftrightarrow x = \frac{1}{50}$$

$$\mathcal{S} = \left\{ \frac{1}{50} \right\}$$

$-36x + 4 - 18x + 18 = -64x - 8$

$$\Leftrightarrow 9x = -18 \quad \Leftrightarrow 10x = -30$$

$$\mathcal{S} = \{-3\}$$

$x^2 - 5x - 6 + 2x^2 - 7x - 15 = -6$

$$\Leftrightarrow 3x^2 - 12x - 15 = 0$$

$$\Leftrightarrow 3(x^2 - 4x - 5) = 0$$

$$\Leftrightarrow 3(x - 5)(x + 1) = 0$$

$$\mathcal{S} = \{-1; 5\}$$

$(3x - 5)(2x - 5) = 0$

$$\mathcal{S} = \left\{ \frac{5}{3}; \frac{5}{2} \right\}$$

Exercice 2.

a) $ED = \mathbb{R} - \{-1; 3\}$

$$(x+4)(x+1) - (x+5)(x-3) = x+9$$

$$\Leftrightarrow x^2 + 5x + 4 - x^2 - 2x + 15 = x + 9$$

$$\Leftrightarrow 2x = -10 \quad \Leftrightarrow x = -5$$

$$\mathcal{S} = \{-5\}$$

b) $ED = \mathbb{R}^* - \{4\}$

$$(x+5)(x-4) - x(x-8) = x+20$$

$$\Leftrightarrow x^2 + x - 20 - x^2 + 8x = x + 20$$

$$\Leftrightarrow 8x = 40 \quad \Leftrightarrow x = 5$$

$$\mathcal{S} = \{5\}$$

$ED = \mathbb{R} - \{-1; 4\}$

$$(x+5)(x+1) - (x-2)(x-4) = 9$$

$$\Leftrightarrow x^2 + 6x + 5 - x^2 + 6x - 8 = 9$$

$$\Leftrightarrow 12x = 12 \quad \Leftrightarrow x = 1$$

$$\mathcal{S} = \{1\}$$

$ED = \mathbb{R}^* - \{6\}$

$$(x+3)(x-6) - x(x-7) = x-3$$

$$\Leftrightarrow x^2 - 3x - 18 - x^2 + 7x = x - 3$$

$$\Leftrightarrow 3x = 15 \quad \Leftrightarrow x = 5$$

$$\mathcal{S} = \{5\}$$

Exercice 3.

$(ED_1 = [-\frac{4}{5}; +\infty[)$

$x - 16 = \sqrt{5x+4} \quad (ED_2 = [16; +\infty[)$

$(ED = [16; +\infty[)$

$$\Leftrightarrow x^2 - 32x + 256 = 5x + 4$$

$$\Leftrightarrow x^2 - 37x + 252 = 0$$

$$\Leftrightarrow (x-9)(x-28) = 0$$

$$\mathcal{S} = \{28\} \quad (9 \text{ sol. à élim.})$$

Vérifications :

si $x = 9 \Rightarrow 9 - 7 \neq 16 \quad \times$

si $x = 28 \Rightarrow 28 - 12 = 16 \quad \checkmark$

$(ED_1 = [-\frac{1}{3}; +\infty[)$

$x - 13 = \sqrt{3x+1} \quad (ED_2 = [13; +\infty[)$

$(ED = [13; +\infty[)$

$$\Leftrightarrow x^2 - 26x + 169 = 3x + 1$$

$$\Leftrightarrow x^2 - 29x + 168 = 0$$

$$\Leftrightarrow (x-8)(x-21) = 0$$

$$\mathcal{S} = \{21\} \quad (8 \text{ sol. à élim.})$$

Vérifications :

si $x = 8 \Rightarrow 8 - 5 \neq 13 \quad \times$

si $x = 21 \Rightarrow 21 - 8 = 13 \quad \checkmark$

Exercice 4.

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0 + \left(\frac{b}{2a}\right)^2 \Leftrightarrow x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\Leftrightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\text{Posons } \Delta = b^2 - 4ac \Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{\Delta}{4a^2}$$

1^{er} cas : si $\Delta > 0$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{\Delta}}{2a} \Leftrightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{\Delta}}{2a} = \frac{-b \pm \sqrt{\Delta}}{2a} \text{ (2 solutions)}$$

2^e cas : si $\Delta = 0$

$$x + \frac{b}{2a} = 0 \Leftrightarrow x = -\frac{b}{2a} \text{ (1 solution double)}$$

3^e cas : si $\Delta < 0$

$$\underbrace{\left(x + \frac{b}{2a}\right)^2}_{>0} = \frac{\Delta}{4a^2} < 0 \Rightarrow \text{pas possible et aucune solution dans } \mathbb{R}$$