

Factorisation et division polynomiale

Exercice 1.

a) $(x + 4)(x - 10)$

b) $(5x - 2y + 9)(5x - 2y - 9)$

c) $(6x + 5)(6x - 5)$

d) $(2x + 1)(x - 4)$

e) $(x^2 - 49)(x^2 - 1) =$
 $(x + 7)(x - 7)(x + 1)(x - 1)$

f) $x^2(2x - y) - 9(2x - y) = (2x - y)(x^2 - 9)$
 $= (2x - y)(x + 3)(x - 3)$

$(x + 5)(x - 8)$

$(x^2 - 4)(x^2 - 1) =$
 $(x + 2)(x - 2)(x + 1)(x - 1)$

$(3x + 1)(x - 3)$

$(7x + 8)(7x - 8)$

$(2x + y + 2)(2x + y - 2)$

$x^2(3x - z) - 4(3x - z) = (3x - z)(x^2 - 4)$
 $= (3x - z)(x + 2)(x - 2)$

Exercice 2.

$$\begin{array}{r}
 a) \quad 2x^4 \quad - 6x^2 + x - 7 = (x^2 + x + 2)(2x^2 - 2x - 8) + 13x + 9 \\
 - 2x^4 - 2x^3 - 4x^2 \\
 \hline
 \quad - 2x^3 - 10x^2 + x \\
 \quad 2x^3 + 2x^2 + 4x \\
 \hline
 \quad \quad - 8x^2 + 5x - 7 \\
 \quad \quad 8x^2 + 8x + 16 \\
 \hline
 \quad \quad \quad 13x + 9
 \end{array}$$

$$\begin{array}{r}
 \dots\dots\dots \\
 3x^4 \quad - 7x^2 + 3x - 9 = (x^2 + 2x + 1)(3x^2 - 6x + 2) + 5x - 11 \\
 - 3x^4 - 6x^3 - 3x^2 \\
 \hline
 \quad - 6x^3 - 10x^2 + 3x \\
 \quad 6x^3 + 12x^2 + 6x \\
 \hline
 \quad \quad 2x^2 + 9x - 9 \\
 \quad \quad - 2x^2 - 4x - 2 \\
 \hline
 \quad \quad \quad 5x - 11
 \end{array}$$

$$\begin{array}{r}
 \text{b) } -x^6 \quad + 5x^4 + 3x^3 \quad + 12 = (x-2)(-x^5 - 2x^4 + x^3 + 5x^2 + 10x + 20) + 52 \\
 \underline{x^6 - 2x^5} \\
 -2x^5 + 5x^4 \\
 \underline{2x^5 - 4x^4} \\
 x^4 + 3x^3 \\
 \underline{-x^4 + 2x^3} \\
 5x^3 \\
 \underline{-5x^3 + 10x^2} \\
 10x^2 \\
 \underline{-10x^2 + 20x} \\
 20x + 12 \\
 \underline{-20x + 40} \\
 52
 \end{array}$$

.....

$$\begin{array}{r}
 -x^6 \quad + 26x^3 + 2x^2 \quad + 15 = (x-3)(-x^5 - 3x^4 - 9x^3 - x^2 - x - 3) + 6 \\
 \underline{x^6 - 3x^5} \\
 -3x^5 \\
 \underline{3x^5 - 9x^4} \\
 -9x^4 + 26x^3 \\
 \underline{9x^4 - 27x^3} \\
 -x^3 + 2x^2 \\
 \underline{x^3 - 3x^2} \\
 -x^2 \\
 \underline{x^2 - 3x} \\
 -3x + 15 \\
 \underline{3x - 9} \\
 6
 \end{array}$$

Exercice 3.

$$\text{a) } r = p(1) = 10 + 7 - 8 = \boxed{9}$$

$$\text{b) } p(3) = 36 - 9 + a = 0 \Leftrightarrow a = \boxed{-27}$$

$$r = p(1) = 13 + 8 - 4 = \boxed{17}$$

$$p(2) = 12 - 10 + a = 0 \Leftrightarrow a = \boxed{-2}$$

Exercice 4.

$$4 \left| \begin{array}{cccc|c} 1 & -2 & -13 & 14 & 24 \\ & 4 & 8 & -20 & -24 \\ \hline 1 & 2 & -5 & -6 & 0 \end{array} \right.$$

$$p(x) = (x - 4)(x^3 + 2x^2 - 5x - 6)$$

$$-1 \left| \begin{array}{cccc|c} 1 & 2 & -5 & -6 \\ & -1 & -1 & 6 \\ \hline 1 & 1 & -6 & 0 \end{array} \right.$$

$$p(x) = (x - 4)(x + 1)(x^2 + x - 6)$$

$$p(x) = (x - 4)(x + 1)(x + 3)(x - 2)$$

$$5 \left| \begin{array}{cccc|c} 1 & 1 & -25 & -37 & 60 \\ & 5 & 30 & 25 & -60 \\ \hline 1 & 6 & 5 & -12 & 0 \end{array} \right.$$

$$p(x) = (x - 5)(x^3 + 6x^2 + 5x - 12)$$

$$1 \left| \begin{array}{ccc|c} 1 & 6 & 5 & -12 \\ & 1 & 7 & 12 \\ \hline 1 & 7 & 12 & 0 \end{array} \right.$$

$$p(x) = (x - 5)(x - 1)(x^2 + 7x + 12)$$

$$p(x) = (x - 5)(x - 1)(x + 3)(x + 4)$$