Vecteurs II

Exercice 1.

a) $\dfrac{3+b_1}{2} = 1 \quad \Leftrightarrow \quad b_1 = -1$

$\dfrac{5+b_2}{2} = 2 \quad \Leftrightarrow \quad b_2 = -1 \Rightarrow \boxed{B(-1; -1)}$

$\dfrac{-3+b_1}{2} = -2 \quad \Leftrightarrow \quad b_1 = -1$

$\dfrac{3+b_2}{2} = 0 \quad \Leftrightarrow \quad b_2 = -3 \Rightarrow \boxed{B(-1; -3)}$

b) $N\left(\dfrac{3+7}{2}; \dfrac{5-2}{2}\right) \quad \Rightarrow \boxed{N\left(5; \dfrac{3}{2}\right)}$

$N\left(\dfrac{-3-8}{2}; \dfrac{3-2}{2}\right) \quad \Rightarrow \boxed{N\left(\dfrac{-11}{2}; \dfrac{1}{2}\right)}$

c) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 7-3 \\ -2-5 \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$

$\Rightarrow \|\overrightarrow{AC}\| = \sqrt{4^2 + (-7)^2} = \sqrt{65}\ \text{u}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} 7+1 \\ -2+1 \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$

$\Rightarrow \|\overrightarrow{BC}\| = \sqrt{8^2 + (-1)^2} = \sqrt{65}\ \text{u}$

$\|\overrightarrow{AC}\| = \|\overrightarrow{BC}\| \Rightarrow \boxed{\Delta\ ABC\ \text{isocèle en } C}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} -8+3 \\ -2-3 \end{pmatrix} = \begin{pmatrix} -5 \\ -5 \end{pmatrix}$

$\Rightarrow \|\overrightarrow{AC}\| = \sqrt{(-5)^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}\ \text{u}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} -8+1 \\ -2+3 \end{pmatrix} = \begin{pmatrix} -7 \\ 1 \end{pmatrix}$

$\Rightarrow \|\overrightarrow{BC}\| = \sqrt{(-7)^2 + 1^2} = \sqrt{50} = 5\sqrt{2}\ \text{u}$

$\|\overrightarrow{AC}\| = \|\overrightarrow{BC}\| \Rightarrow \boxed{\Delta\ ABC\ \text{isocèle en } C}$

Exercice 2.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 0+3 \\ 4-0 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 3+3 \\ 8-0 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$\det(\overrightarrow{AB}; \overrightarrow{AC}) = \begin{vmatrix} 3 & 6 \\ 4 & 8 \end{vmatrix} = 24 - 24 = 0$

$\Rightarrow \quad \overrightarrow{AB}\ \text{colin. à } \overrightarrow{AC} \Rightarrow \boxed{A,\ B,\ C\ \text{alignés}}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 0+5 \\ -3-0 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 10+5 \\ -9-0 \end{pmatrix} = \begin{pmatrix} 15 \\ -9 \end{pmatrix}$

$\det(\overrightarrow{AB}; \overrightarrow{AC}) = \begin{vmatrix} 5 & 15 \\ -3 & -9 \end{vmatrix} = -45 - (-45) = 0$

$\Rightarrow \quad \overrightarrow{AB}\ \text{colin. à } \overrightarrow{AC} \Rightarrow \boxed{A,\ B,\ C\ \text{alignés}}$

Exercice 3.

a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 5-1 \\ -4-2 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$ $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 6-2 \\ -4+3 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} 8-5 \\ -2+4 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} 8-6 \\ 4+4 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$

$\overrightarrow{AB} \cdot \overrightarrow{BC} = 12 - 12 = 0 \quad \Leftrightarrow \quad AB \perp BC$ $\overrightarrow{AB} \cdot \overrightarrow{BC} = 8 - 8 = 0 \quad \Leftrightarrow \quad AB \perp BC$

$\Rightarrow \boxed{\Delta\ ABC \text{ rectangle en } B}$ $\Rightarrow \boxed{\Delta\ ABC \text{ rectangle en } B}$

b) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC}$ $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC}$

$\Rightarrow \overrightarrow{OD} = \begin{pmatrix} 1+3 \\ 2+2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad \Rightarrow \boxed{D(4;4)}$ $\Rightarrow \overrightarrow{OD} = \begin{pmatrix} 2+2 \\ -3+8 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \quad \Rightarrow \boxed{D(4;5)}$

c) $\|\overrightarrow{AB}\| = \sqrt{4^2 + (-6)^2} = \sqrt{42} = 2\sqrt{13}$ $\|\overrightarrow{AB}\| = \sqrt{4^2 + (-1)^2} = \sqrt{17}$

$\|\overrightarrow{BC}\| = \sqrt{3^2 + 2^2} = \sqrt{13}$ $\|\overrightarrow{BC}\| = \sqrt{2^2 + 8^2} = \sqrt{68} = 2\sqrt{17}$

$\Rightarrow$ aire de $ABCD = 2\sqrt{13} \cdot \sqrt{13} = \boxed{26 \text{ u}^2}$ $\Rightarrow$ aire de $ABCD = \sqrt{17} \cdot 2\sqrt{17} = \boxed{34 \text{ u}^2}$

Exercice 4.

$\det(\vec{a}; \vec{b}) = \begin{vmatrix} m+2 & m-4 \\ 5 & 2 \end{vmatrix} =$ $\det(\vec{a}; \vec{b}) = \begin{vmatrix} m-6 & m-5 \\ 3 & 2 \end{vmatrix} =$

$2m + 4 - (5m - 20) = -3m + 24$ $2m - 12 - (3m - 15) = -m + 3$

$-3m + 24 = 0 \quad \Leftrightarrow \quad \boxed{m = 8}$ $-m + 3 = 0 \quad \Leftrightarrow \quad \boxed{m = 3}$

$\Rightarrow \vec{a} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ $\Rightarrow \vec{a} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$

$\vec{a} \cdot \vec{c} = 20n - 30 + 5n + 5 = -25n - 25$ $\vec{a} \cdot \vec{c} = -6n + 9 + 3n + 3 = -3n + 12$

$25n - 25 = 0 \quad \Leftrightarrow \quad \boxed{n = 1}$ $-3n + 12 = 0 \quad \Leftrightarrow \quad \boxed{n = 4}$