

Factorisation et division polynomiale

Exercice 1.

$$\frac{3}{2} \left| \begin{array}{cccc} 12 & -28 & 7 & 12 \\ & 18 & -15 & -12 \\ \hline 12 & -10 & -8 & \boxed{0} \end{array} \right.$$

$$\Rightarrow (6x^2 - 5x - 4)(2x - 3)$$

$$\Rightarrow \boxed{(3x - 4)(2x + 1)(2x - 3)}$$

$$\frac{4}{3} \left| \begin{array}{cccc} 6 & -47 & -20 & 96 \\ & 8 & -52 & -96 \\ \hline 6 & -39 & -72 & \boxed{0} \end{array} \right.$$

$$\Rightarrow (3x - 4)(2x^2 - 13x - 24)$$

$$\Rightarrow \boxed{(3x - 4)(2x + 3)(x - 8)}$$

Exercice 2.

$$p(x) = (2x - 1)(x + 5)(x - 3)(ax + b)$$

$$\begin{cases} p(-1) = -192 \\ p(2) = 147 \end{cases}$$

$$\Rightarrow \begin{cases} 48(-a + b) = -192 \\ -21(2a + b) = 147 \end{cases}$$

$$\Rightarrow \begin{cases} -a + b = -4 \\ -2a - b = 7 \end{cases} \Rightarrow -3a = 3$$

$$\Rightarrow a = -1 \Rightarrow b = -4 - 1 = -5$$

$$\Rightarrow \boxed{p(x) = (2x - 1)(x + 5)(x - 3)(-x - 5)}$$

$$p(x) = (2x - 1)(x + 4)(x - 6)(ax + b)$$

$$\begin{cases} p(1) = -75 \\ p(-2) = -960 \end{cases}$$

$$\Rightarrow \begin{cases} -25(a + b) = -75 \\ 80(-2a + b) = -960 \end{cases}$$

$$\Rightarrow \begin{cases} a + b = 3 \\ 2a - b = 12 \end{cases} \Rightarrow 3a = 15$$

$$\Rightarrow a = 5 \Rightarrow b = 3 - 5 = -2$$

$$\Rightarrow \boxed{p(x) = (2x - 1)(x + 4)(x - 6)(5x - 2)}$$

Exercice 3.

$$\begin{aligned} \text{a) } p(m + 2) &= (m + 2)^2 - 2(m + 3)(m + 2) + \\ & m^2 + 6m + 8 = m^2 + 4m + 4 - 2m^2 - 10m - \\ & 12 + m^2 + 6m + 8 = 0 \end{aligned}$$

$$\text{b) } \boxed{p(x) = [x - (m + 2)][x - (m + 4)]}$$

$$\begin{aligned} p(m + 1) &= (m + 1)^2 - 2(m + 2)(m + 1) + \\ & m^2 + 4m + 3 = m^2 + 2m + 1 - 2m^2 - 6m - \\ & 4 + m^2 + 4m + 3 = 0 \end{aligned}$$

$$\boxed{p(x) = [x - (m + 1)][x - (m + 3)]}$$

Exercice 4.

a) $(3x - 2)(x - 1)$

$(2x - 5)(x - 1)$

b) $[(4x - 1) + 4][(4x - 1) - 4] = (4x + 3)(4x - 5)$

$[(5x + 1) + 5][(5x + 1) - 5] = (5x + 6)(5x - 4)$

c) $(x^2 + 9)(x^2 - 9) = (x^2 + 9)(x + 3)(x - 3)$

$(x^2 + 4)(x^2 - 4) = (x^2 + 4)(x + 2)(x - 2)$

$$1 \left| \begin{array}{cccc} 6 & 1 & -10 & 3 \\ & 6 & 7 & -3 \\ \hline 6 & 7 & -3 & 0 \end{array} \right.$$

$$1 \left| \begin{array}{cccc} 10 & -11 & -1 & 2 \\ & 10 & -1 & -2 \\ \hline 10 & -1 & -2 & 0 \end{array} \right.$$

d) $(6x^2 + 7x - 3)(x - 1) =$

$(3x - 1)(2x + 3)(x - 1)$

$(10x^2 - x - 2)(x - 1) =$

$(5x + 2)(2x - 1)(x - 1)$