Vecteurs II

Exercice 1.

a)

$$\overrightarrow{AB}=\begin{pmatrix}2\\-1\\-1\end{pmatrix}\ \overrightarrow{AD}=\begin{pmatrix}1\\5\\-1\end{pmatrix}\ \overrightarrow{AE}=\begin{pmatrix}3\\1\\3\end{pmatrix}$$

$$\det(\overrightarrow{AB};\overrightarrow{AD};\overrightarrow{AE})=\begin{vmatrix}2&1&3\\-1&5&1\\-1&-1&3\end{vmatrix}=$$

$$2(15+1)-(-1)(3+3)+(-1)(1-15)=$$

$$32+6+14=52\quad\Rightarrow\quad V=\boxed{52\ \text{u}^3}$$

b) $$\overrightarrow{BC}=\overrightarrow{AD}=\begin{pmatrix}1\\5\\-1\end{pmatrix}$$

$$\overrightarrow{OC}=\overrightarrow{AD}+\overrightarrow{OB}=\begin{pmatrix}3\\5\\-1\end{pmatrix}$$

$$\Rightarrow\ \boxed{C(3;5;-1)}$$

c) $$\begin{cases}\vec{n}\cdot\overrightarrow{AB}=0\\ \vec{n}\cdot\overrightarrow{AD}=0\end{cases}\quad\Rightarrow\quad\begin{cases}2x-y-11=0\\ x+5y-11=0\end{cases}$$

$$\Rightarrow\ \begin{cases}2x-y=11\\ -2x-10y=-22\\ \hline -11y=-11\end{cases}$$

$$\Rightarrow\ \boxed{y=1\qquad x=6}$$

d) $$\overrightarrow{AB}\times\overrightarrow{AD}=\begin{pmatrix}1+5\\2-1\\10+1\end{pmatrix}=\begin{pmatrix}6\\1\\11\end{pmatrix}$$

$$\Rightarrow\ \|\overrightarrow{AB}\times\overrightarrow{AD}\|=\sqrt{36+1+121}=\sqrt{158}$$

$$\Rightarrow\quad \sigma_{ABD}=\boxed{\frac{\sqrt{158}}{2}\ \text{u}^2}$$

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a)

$$\overrightarrow{AB}=\begin{pmatrix}-1\\-1\\3\end{pmatrix}\ \overrightarrow{AD}=\begin{pmatrix}1\\4\\0\end{pmatrix}\ \overrightarrow{AE}=\begin{pmatrix}0\\2\\5\end{pmatrix}$$

$$\det(\overrightarrow{AB};\overrightarrow{AD};\overrightarrow{AE})=\begin{vmatrix}-1&1&0\\-1&4&2\\3&0&5\end{vmatrix}=$$

$$-1(20-0)-(-1)(5-0)+3(2-0)=$$

$$-20+5+6=-9\quad\Rightarrow\quad V=\boxed{9\ \text{u}^3}$$

b) $$\overrightarrow{BC}=\overrightarrow{AD}=\begin{pmatrix}1\\4\\0\end{pmatrix}$$

$$\overrightarrow{OC}=\overrightarrow{AD}+\overrightarrow{OB}=\begin{pmatrix}1\\4\\3\end{pmatrix}$$

$$\Rightarrow\ \boxed{C(1;4;3)}$$

c) $$\begin{cases}\vec{n}\cdot\overrightarrow{AB}=0\\ \vec{n}\cdot\overrightarrow{AD}=0\end{cases}\quad\Rightarrow\quad\begin{cases}-x-y+9=0\\ x+4y=0\end{cases}$$

$$\Rightarrow\ \begin{cases}-x-y=-9\\ x+4y=0\\ \hline 3y=-9\end{cases}$$

$$\Rightarrow\ \boxed{y=-3\qquad x=12}$$

d) $$\overrightarrow{AB}\times\overrightarrow{AD}=\begin{pmatrix}0-12\\0+3\\-4+1\end{pmatrix}=\begin{pmatrix}-12\\3\\-3\end{pmatrix}$$

$$\Rightarrow\ \|\overrightarrow{AB}\times\overrightarrow{AD}\|=\sqrt{144+9+9}=\sqrt{162}$$

$$\Rightarrow\quad \sigma_{ABD}=\frac{1}{2}\sqrt{162}=\boxed{\frac{9\sqrt{2}}{2}\ \text{u}^2}$$

e) $\overrightarrow{AE} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$ et $\vec{n} = \begin{pmatrix} 6 \\ 1 \\ 11 \end{pmatrix}$

$$\cos(\beta) = \frac{18+1+33}{\sqrt{9+1+9}\sqrt{36+1+121}}$$

$$= \frac{52}{\sqrt{19}\sqrt{158}} \quad \Rightarrow \quad \beta \simeq 18,37°$$

$$\Rightarrow \quad \alpha = 90 - \beta \simeq \boxed{71,64°}$$

$\overrightarrow{AE} = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix}$ et $\vec{n} = \begin{pmatrix} 12 \\ -3 \\ 3 \end{pmatrix}$

$$\cos(\beta) = \frac{0-6+15}{\sqrt{0+4+25}\sqrt{144+9+9}}$$

$$= \frac{9}{\sqrt{29}\sqrt{162}} \quad \Rightarrow \quad \beta \simeq 82,45°$$

$$\Rightarrow \quad \alpha = 90 - \beta \simeq \boxed{7,55°}$$

Exercice 2.

a) $C\,(3 \cdot 5 - 4 - 1; 3 \cdot 2 - 5 - 2)$

$\Rightarrow \boxed{C(10; -1)}$

b) $\overrightarrow{AB} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}$ $\quad \overrightarrow{BC} = \begin{pmatrix} 9 \\ -3 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 6 \\ -6 \end{pmatrix}$

$\|\overrightarrow{AB}\| = \sqrt{18} = 3\sqrt{2}$

$\|\overrightarrow{BC}\| = \sqrt{90} = 3\sqrt{10}$

$\|\overrightarrow{AC}\| = \sqrt{72} = 6\sqrt{2}$

$\Rightarrow \boxed{\text{périmètre du } \Delta\ ABC = 9\sqrt{2} + 3\sqrt{10} \text{ u}}$

c) $\overrightarrow{AB} \cdot \overrightarrow{AC} = -18 + 18 = 0$

$\Rightarrow \quad \Delta\ ABC$ est rectangle en A

d) $\sigma_{ABC} = \frac{1}{2} \cdot 3\sqrt{2} \cdot 6\sqrt{2} = \boxed{18 \text{ u}^2}$

e) I milieu de BC (cercle de Thalès)

$\Rightarrow \boxed{I(5.5; 0.5)}$

$C\,(3 \cdot 4 - 7 - 4; 3 \cdot 1 - 2 - 5)$

$\Rightarrow \boxed{C(1; -4)}$

$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$ $\quad \overrightarrow{BC} = \begin{pmatrix} -3 \\ -9 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} -6 \\ -6 \end{pmatrix}$

$\|\overrightarrow{AB}\| = \sqrt{18} = 3\sqrt{2}$

$\|\overrightarrow{BC}\| = \sqrt{90} = 3\sqrt{10}$

$\|\overrightarrow{AC}\| = \sqrt{72} = 6\sqrt{2}$

$\Rightarrow \boxed{\text{périmètre du } \Delta\ ABC = 9\sqrt{2} + 3\sqrt{10} \text{ u}}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 18 - 18 = 0$

$\Rightarrow \quad \Delta\ ABC$ est rectangle en A

$\sigma_{ABC} = \frac{1}{2} \cdot 3\sqrt{2} \cdot 6\sqrt{2} = \boxed{18 \text{ u}^2}$

I milieu de BC (cercle de Thalès)

$\Rightarrow \boxed{I(2.5; 0.5)}$

Exercice 3.

$$\overrightarrow{AB} = \begin{pmatrix} -3 \\ -1 \\ 8 \end{pmatrix} \qquad \overrightarrow{AC} = \begin{pmatrix} 12 \\ 4 \\ k+2 \end{pmatrix}$$

$$\Rightarrow \quad \overrightarrow{AC} = -4\overrightarrow{AB}$$

$$\Rightarrow \quad k+2 = -32 \quad \Rightarrow \boxed{k = -34}$$

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \qquad \overrightarrow{AC} = \begin{pmatrix} -8 \\ 4 \\ k-3 \end{pmatrix}$$

$$\Rightarrow \quad \overrightarrow{AC} = -4\overrightarrow{AB}$$

$$\Rightarrow \quad k-3 = -8 \quad \Rightarrow \boxed{k = -5}$$

Exercice 4.

$$\det(\vec{u};\vec{v};\vec{w}) = \begin{vmatrix} 3 & 2 & 2 \\ 1 & k & -1 \\ 2 & -1 & k \end{vmatrix} =$$

$$3(k^2-1) - 1(2k+2) + 2(-2-2k) =$$

$$3k^2 - 3 - 2k - 2 - 4 - 4k =$$

$$3k^2 - 6k - 9 = 0 \quad \Rightarrow \quad k^2 - 2k - 3 = 0$$

$$\Rightarrow \quad (k-3)(k+1) = 0$$

$$\Rightarrow \quad \boxed{k_1 = -1 \qquad k_2 = 3}$$

$$\det(\vec{u};\vec{v};\vec{w}) = \begin{vmatrix} 1 & 4 & -2 \\ 0 & 3 & k \\ -2 & k & -1 \end{vmatrix} =$$

$$1(-3-k^2) - 0(-4+2k) + (-2)(4k+6) =$$

$$-3 - k^2 - 8k - 12 =$$

$$-k^2 - 8k - 15 = 0 \quad \Rightarrow \quad k^2 + 8k + 15 = 0$$

$$\Rightarrow \quad (k+3)(k+5) = 0$$

$$\Rightarrow \quad \boxed{k_1 = -5 \qquad k_2 = -3}$$