

## Vecteurs I

### Exercice 1.

$$\begin{cases} 5k - 2m = 5 \\ 3k - m = 10 \end{cases}$$

$$\begin{array}{r} 5k - 2m = 5 \\ -6k + 2m = -20 \\ \hline -k = -15 \end{array}$$

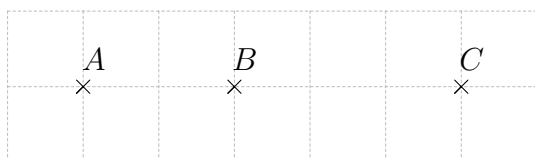
$$\Rightarrow k = \boxed{15} \text{ et } m = \boxed{35}$$

$$\begin{cases} 2k - 3m = 4 \\ 5k - 6m = 13 \end{cases}$$

$$\begin{array}{r} -4k + 6m = -8 \\ 5k - 6m = 13 \\ \hline k = 5 \end{array}$$

$$\Rightarrow k = \boxed{5} \text{ et } m = \boxed{2}$$

### Exercice 2.



### Exercice 3.

$$\begin{aligned} \text{a) } \det(\vec{a}; \vec{b}; \vec{c}) &= 9 - 30 - 1 - (3 - 90 - 1) \\ &= -22 - (-88) = 66 \neq 0 \end{aligned}$$

$$\begin{aligned} \det(\vec{a}; \vec{b}; \vec{c}) &= -2 + 15 - 6 - (10 - 18 - 1) \\ &= 7 - (-9) = 16 \neq 0 \end{aligned}$$

b)

$$\begin{cases} 3x - y + z = 29 \\ x + 3y + 30z = 6 \\ x - y + z = 17 \end{cases}$$

$$\begin{array}{r} 3x - y + z = 29 \\ -x + y - z = -17 \\ \hline 2x = 12 \end{array}$$

$$\Rightarrow x = 6 \Rightarrow \begin{cases} -y + z = 11 \\ 3y + 30z = 0 \end{cases}$$

$$\begin{array}{r} -y + z = 11 \\ y + 10z = 0 \\ \hline 11z = 11 \end{array} \Rightarrow z = 1$$

$$\Rightarrow y = -10$$

$$\Rightarrow \boxed{6\vec{a} - 10\vec{b} + \vec{c} = \vec{v}}$$

$$\begin{cases} x - y - z = 6 \\ x - 2y - 3z = 10 \\ 5x + 6y + z = 2 \end{cases}$$

$$\begin{array}{r} 3x - 3y - 3z = 18 \\ -x + 2y + 3z = -10 \\ \hline 2x - y = 8 \end{array}$$

$$\begin{array}{r} x - y - z = 6 \\ 5x + 6y + z = 2 \\ \hline 6x + 5y = 8 \end{array}$$

$$\begin{array}{r} 10x - 5y = 40 \\ 6x + 5y = 8 \\ \hline 16x = 48 \end{array} \Rightarrow x = 3$$

$$\Rightarrow y = -2 \Rightarrow z = -1$$

$$\Rightarrow \boxed{3\vec{a} - 2\vec{b} - \vec{c} = \vec{v}}$$

**Exercice 4.**

$$\text{a) } \vec{IJ} = \frac{1}{2} \underbrace{(\vec{AB} + \vec{BF})}_{\vec{AF}} = \boxed{\frac{1}{2} \vec{AF}}$$

$$\text{b) } \vec{IJ} = \boxed{\begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix}}$$

$$\vec{IN} = \boxed{\begin{pmatrix} -1/2 \\ 1/2 \\ 1/2 \end{pmatrix}}$$

$$\vec{MN} = \boxed{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}$$

$$\vec{LN} = \frac{1}{2} \underbrace{(\vec{AB} + \vec{BC})}_{\vec{AC}} = \boxed{\frac{1}{2} \vec{AC}}$$

$$\vec{LI} = \boxed{\begin{pmatrix} 1/2 \\ -1/2 \\ 0 \end{pmatrix}}$$

$$\vec{NL} = \boxed{\begin{pmatrix} -1/2 \\ 1/2 \\ -1/2 \end{pmatrix}}$$

$$\vec{MN} = \boxed{\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}}$$

**Exercice 5.**

$$\vec{AB} = 5\vec{XA} + 6 \underbrace{\vec{XB}}_{\vec{XA} + \vec{AB}}$$

$$\Rightarrow \vec{AB} = 5\vec{XA} + 6\vec{XA} + 6\vec{AB}$$

$$\Rightarrow -5\vec{AB} = -11\vec{AX}$$

$$\Rightarrow \boxed{\vec{AX} = \frac{5}{11} \vec{AB}}$$

$$\vec{AB} = 7\vec{XA} + 8 \underbrace{\vec{XB}}_{\vec{XA} + \vec{AB}}$$

$$\Rightarrow \vec{AB} = 7\vec{XA} + 8\vec{XA} + 8\vec{AB}$$

$$\Rightarrow -7\vec{AB} = -15\vec{AX}$$

$$\Rightarrow \boxed{\vec{AX} = \frac{7}{15} \vec{AB}}$$