Asymptotes

2.6.1

a) $f(x) = \dfrac{x^2+x-2}{x^2+2x-3} = \dfrac{(x+2)(x-1)}{(x+3)(x-1)} \quad \Rightarrow \quad ED(f) = \mathbb{R}\backslash\{-3;1\}$

$$\lim_{x\to -3} \frac{x^2+x-2}{x^2+2x-3} \overbrace{=}^{\frac{4}{0}} \begin{array}{ll} < & +\infty \\ > & -\infty \end{array} \qquad \Rightarrow \quad \text{AV} : x = -3$$

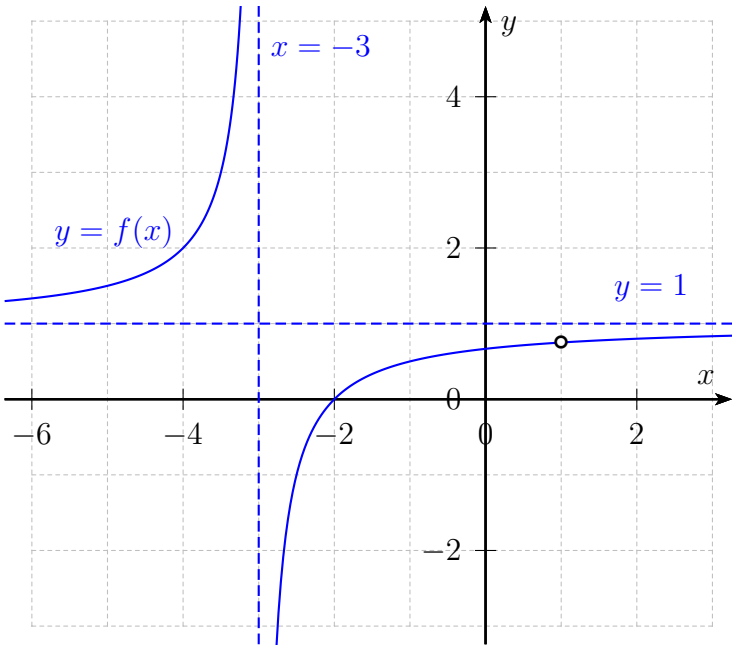
$$\lim_{x\to 1} \frac{x^2+x-2}{x^2+2x-3} \overbrace{=}^{\frac{0}{0}\ \text{fi}} \lim_{x\to 1} \frac{(x+2)(x-1)}{(x+3)(x-1)} = \lim_{x\to 1} \frac{x+2}{x+3} = \frac{3}{4}$$

$$\Rightarrow \quad \text{trou} : \left(1; \frac{3}{4}\right)$$

$$\lim_{x\to\infty} \frac{x^2+x-2}{x^2+2x-3} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1 \quad \Rightarrow \text{AH} : y = 1$$

$$\delta(x) = \frac{x^2+x-2}{x^2+2x-3} - 1 = \frac{-x+1}{x^2+2x-3}$$

x	$-\infty$	-3	1	$+\infty$
$\delta(x)$	$+$	$-$	$-$	
position	dessus	dessous	dessous	



b) $f(x) = \dfrac{x^2+x+1}{x-2} \quad \Rightarrow \quad ED(f) = \mathbb{R}\backslash\{2\}$

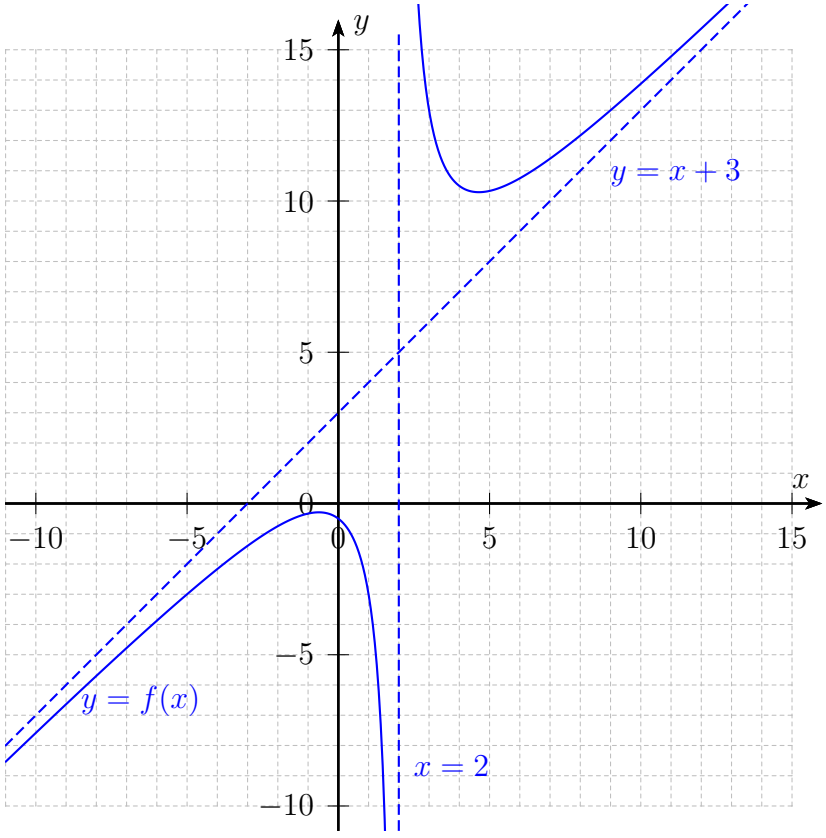
$$\lim_{x\to 2} \frac{x^2+x+1}{x-2} \overbrace{=}^{\frac{7}{0}} \begin{array}{ll} < & -\infty \\ > & +\infty \end{array} \qquad \Rightarrow \quad \text{AV} : x = 2$$

$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1$$

$$\lim_{x\to\infty} (f(x) - x) = \lim_{x\to\infty} \frac{3x+1}{x-2} = \lim_{x\to\infty} \frac{3x}{x} = 3 \quad \Rightarrow \quad \text{AO} : y = x+3$$

$$\delta(x) = \frac{x^2+x+1}{x-2} - (x+3) = \frac{7}{x-2}$$

x	$-\infty$	2	$+\infty$
$\delta(x)$	$-$		$+$
position	dessous		dessus



c) $f(x) = \dfrac{2x^2+6x}{(x+2)^2} \quad \Rightarrow \quad ED(f) = \mathbb{R}\backslash\{-2\}$

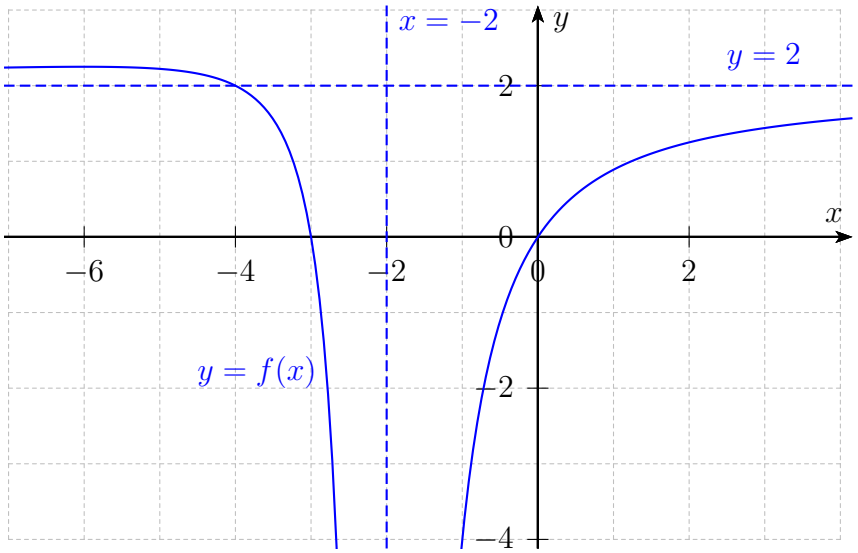
$$\lim_{x\to -2} \frac{2x^2+6x}{(x+2)^2} \overbrace{=}^{\frac{-4}{0_+}} -\infty \quad \Rightarrow \quad \text{AV} : x = -2$$

$$\lim_{x\to \infty} \frac{2x^2+6x}{(x+2)^2} = \lim_{x\to \infty} \frac{2x^2}{x^2} = 2 \quad \Rightarrow \quad \text{AH} : y = 2$$

$$\delta(x) = \frac{2x^2+6x}{(x+2)^2} - 2 = \frac{-2x-8}{(x+2)^2}$$

x	$-\infty$	-4	-2	$+\infty$
$\delta(x)$	$+$	0	$-$ ‖	$-$
position	dessus	$\cap$	dessous ‖	dessous

intersection avec l'AH : $(-4;2)$



d) $f(x) = \dfrac{x^3}{4-x^2} = \dfrac{x^3}{(2-x)(2+x)} \quad \Rightarrow \quad ED(f) = \mathbb{R}\backslash\{-2;2\}$

$$\lim_{x\to -2} \frac{x^3}{4-x^2} \overbrace{=}^{\frac{-8}{0}} \begin{array}{ll} < & +\infty \\ > & -\infty \end{array} \quad \Rightarrow \quad \text{AV} : x = -2$$

$$\lim_{x\to 2} \frac{x^3}{4-x^2} \overbrace{=}^{\frac{8}{0}} \begin{array}{ll} < & +\infty \\ > & -\infty \end{array} \quad \Rightarrow \quad \text{AV} : x = 2$$

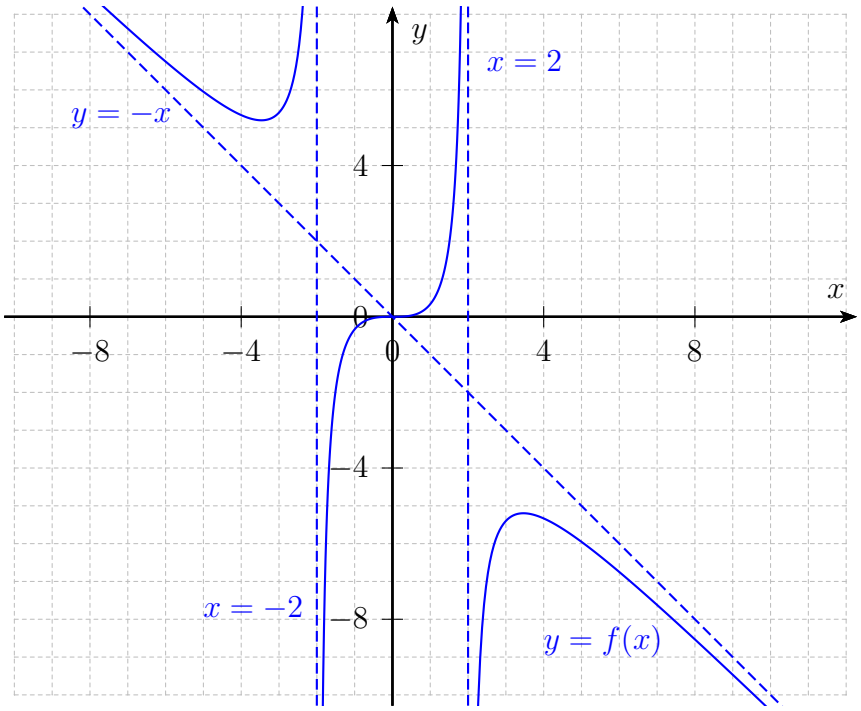
$$\lim_{x\to \infty} \frac{f(x)}{x} = \lim_{x\to \infty} \frac{x^3}{-x^3} = -1$$

$$\lim_{x\to\infty}(f(x)+x)=\lim_{x\to\infty}\frac{4x}{4-x^2}=\lim_{x\to\infty}\frac{4x}{-x^2}=0 \quad\Rightarrow\quad \text{AO}: y=-x$$

$$\delta(x)=\frac{x^3}{4-x^2}+x=\frac{4x}{4-x^2}$$

x	$-\infty$	-2	0	-2	$+\infty$
$\delta(x)$	$+$	$-$	0	$+$	$-$
position	dessus	dessous	$\cap$	dessus	dessous

inter. avec l'AH : $(0;0)$



e) $f(x)=\dfrac{x^2-4x}{x^2-4x+3}=\dfrac{x(x-4)}{(x-3)(x-1)} \quad\Rightarrow\quad ED(f)=\mathbb{R}\backslash\{1;3\}$

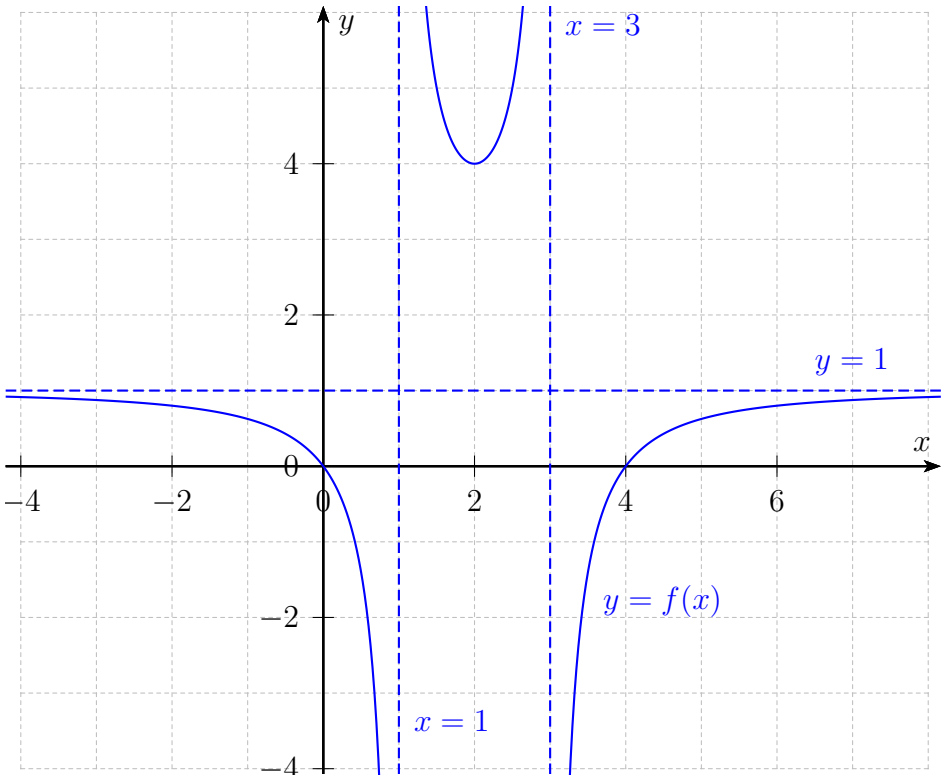
$$\lim_{x\to1}\frac{x^2-4x}{x^2-4x+3}\overset{\frac{-3}{0}}{=}\begin{cases}<&-\infty\\>&+\infty\end{cases} \quad\Rightarrow\quad \text{AV}: x=1$$

$$\lim_{x\to3}\frac{x^2-4x}{x^2-4x+3}\overset{\frac{-3}{0}}{=}\begin{cases}<&+\infty\\>&-\infty\end{cases} \quad\Rightarrow\quad \text{AV}: x=3$$

$$\lim_{x\to\infty}\frac{x^2-4x}{x^2-4x+3}=\lim_{x\to\infty}\frac{x^2}{x^2}=1 \quad\Rightarrow\quad \text{AH}: y=1$$

$$\delta(x)=\frac{x^2-4x}{x^2-4x+3}-1=\frac{-3}{x^2-4x+3}$$

x	$-\infty$	1	3	$+\infty$
$\delta(x)$	$-$	$+$	$-$	
position	dessous	dessus	dessous	



f) $f(x) = \dfrac{x^3 - 3x^2 + 2}{x^2 - 4} = \dfrac{x^3 - 3x^2 + 2}{(x-2)(x+2)} \quad \Rightarrow \quad ED(f) = \mathbb{R}\backslash\{-2;2\}$

$$\lim_{x\to -2} \frac{x^3 - 3x^2 + 2}{x^2 - 4} \overset{\frac{-18}{0}}{=} \begin{matrix} < & -\infty \\ > & +\infty \end{matrix} \quad \Rightarrow \quad \text{AV} : x = -2$$

$$\lim_{x\to 2} \frac{x^3 - 3x^2 + 2}{x^2 - 4} \overset{\frac{-2}{0}}{=} \begin{matrix} < & +\infty \\ > & -\infty \end{matrix} \quad \Rightarrow \quad \text{AV} : x = 2$$

$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^3}{x^3} = 1$$

$$\lim_{x\to\infty} (f(x) - x) = \lim_{x\to\infty} \frac{-3x^2 + 6}{x^2 - 4} = \lim_{x\to\infty} \frac{-3x^2}{x^2} = -3 \quad \Rightarrow \quad \text{AO} : y = x - 3$$

$$\delta(x) = \frac{x^3 - 3x^2 + 2}{x^2 - 4} - (x - 3) = \frac{4x - 10}{x^2 - 4}$$

x	$-\infty$	-2	2	$5/2$	$+\infty$
$\delta(x)$	$-$	$+$	$-$	0	$+$
position	dessous	dessus	dessous	$\cap$	dessus

inter. avec l'AH : $\left(\frac{5}{2}; -\frac{1}{2}\right)$