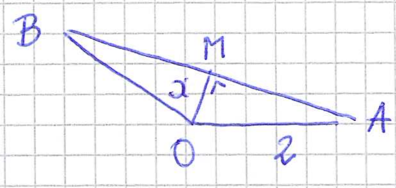
1

a) 

Pythagore: $AM = \sqrt{4-x^2}$

$A(x) = x\cdot\sqrt{4-x^2}$ $\qquad x \in \,]0;2[$

b) $A'(x) = 4-x^2 + x\cdot\frac{(-\cancel{2}x)}{\cancel{2}\sqrt{4-x^2}} = \frac{4-x^2-x^2}{\sqrt{4-x^2}} = \frac{4-2x^2}{\sqrt{4-x^2}}$

c)

x	$-\infty$	0		$\sqrt{2}$		2	$+\infty$
$A'(x)$			$+$	0	$-$		
$A(x)$			$\nearrow$	2	$\searrow$		

$\Rightarrow \quad x = \sqrt{2}$ m

d) $A(\sqrt{2}) = 2$ m^2

2

a) $h = \frac{15'360}{30x} = \frac{512}{x}$

$f(x) = x\cdot\frac{512}{x}\cdot\frac{1}{10} + \left(x\cdot\frac{512}{x} + 2\cdot 30\cdot\frac{512}{x} + 30x\right)\cdot\frac{1}{20}$

$= \frac{512}{10} + \frac{256}{10} + \frac{15'360}{10x} + \frac{15x}{10} = \frac{15x^2+768x+15'360}{10x}$

$x \in \,]0;+\infty[$

b) $f'(x) = \frac{(30x+768)10x - (15x^2+768x+15'360)\cdot 10}{100x^2}$

$= \frac{300x^2 + \cancel{7'680x} - 150x^2 - \cancel{7'680x} - 153'600}{100x^2} = \frac{150x^2 - 153'600}{100x^2}$

$= \frac{3(x^2-1024)}{2x^2} = \frac{3(x-32)(x+32)}{2x^2}$

x	$-\infty$	0		32		$+\infty$
$f'(x)$	//////	‖	$-$	0	$+$	
$f(x)$	//////	‖	$\searrow$	172,8	$\nearrow$	

dim. : $x = 32$ cm $h = 16$ cm et 30 cm de profondeur

c) $f(32) = 172,80$ frs (coût minimal)

③

a) $l = \dfrac{3750}{\pi r^2} \cdot 2 = \dfrac{7500}{\pi r^2}$

$$C(r) = \pi r^2 \cdot 35 + \cancel{2}\pi \cancel{r} \cdot \frac{7500}{\cancel{\pi} r^{\cancel{2}}} \cdot 15 = 35\pi r^2 + \frac{112500}{r}$$

$$= \frac{35\pi r^3 + 112500}{r} \qquad r \in]0; +\infty[$$

b) $$C'(r) = \frac{105\pi r^2 \cdot r - (35\pi r^3 + 112500) \cdot 1}{r^2} = \frac{105\pi r^3 - 35\pi r^3 - 112500}{r^2}$$

$$= \frac{70\pi r^3 - 112500}{r^2} = \frac{10\,(7\pi r^3 - 11250)}{r^2}$$

zéro : $r = \sqrt[3]{\dfrac{11250}{7\pi}} \cong 7,997 \Rightarrow 8$ m

r	$-\infty$	0		8		$+\infty$
$C'(r)$	//////	‖	$-$	0	$+$	
$C(r)$	//////	‖	$\searrow$	~ 21100	$\nearrow$	

$r \cong 8$m $l \cong 37,32$ m

$C(8) \cong 21'100$ frs (coût minimal)

(4)

a) $h = \frac{36}{2x^2} = \frac{18}{x^2}$

$A(x) = 2\cancel{x} \cdot \frac{18}{x^{\cancel{2}}} + 2 \cdot 2\cancel{x} \cdot \frac{18}{x^{\cancel{2}}} + 2x^2 = \frac{108}{x} + 2x^2 = \frac{2x^3 + 108}{x}$

$x \in \,]0 ; +\infty[$

b) $A'(x) = \frac{6x^2 \cdot x - (2x^3+108) \cdot 1}{x^2} = \frac{4x^3 - 108}{x^2} = \frac{4(x^3-27)}{x^2} = \frac{4(x-3)(x^2+3x+9)}{x^2}$

x	$-\infty$	0		3		$+\infty$
$A'(x)$	////	‖	$-$	0	$+$	
$A(x)$	////	‖	$\searrow$	54	$\nearrow$	

min. : $x = 3$

(5) a) longueur du jardin : $\frac{2000}{x}$

$V(x) = 0{,}8 \cdot x \cdot 1 \cdot 2 + 0{,}8 \cdot 1 \cdot 2{,}5 \cdot 2 + 0{,}8 \cdot \frac{2000}{x} \cdot 2{,}5$

$= 1{,}6x + 4 + \frac{4000}{x} = \frac{1{,}6x^2 + 4x + 4000}{x}$

$x \in \,]0 ; +\infty[$

b) $V'(x) = \frac{(3{,}2x+4) \cdot x - (1{,}6x^2+4x+4000) \cdot 1}{x^2} = \frac{1{,}6x^2 - 4000}{x^2}$

$= \frac{1{,}6\,(x^2 - 2500)}{x^2} = \frac{1{,}6\,(x-50)(x+50)}{x^2}$

x	$-\infty$	0		50		$+\infty$
$V'(x)$	////	‖	$-$	0	$+$	
$V(x)$	////	‖	$\searrow$	164	$\nearrow$	

$\Rightarrow$ $x = 50$ m $\Rightarrow$ longueur : 40 m

$V(50) = 164$ m³ (volume minimal)

⑥

a) temps : $\frac{1500}{v}$

$$\Rightarrow P(v) = 26 \cdot \frac{1500}{v} + \left(\frac{600}{v} + \frac{v}{3}\right) \cdot 15 \cdot 2 = \frac{39'000}{v} + \frac{18'000}{v} + 10v$$

$$= \frac{57'000}{v} + 10v$$

$v \in \,]0; +\infty[$

b) $$P'(v) = -\frac{57'000}{v^2} + 10 = \frac{10v^2 - 57'000}{v^2} = \frac{10\,(v^2 - 5'700)}{v^2}$$

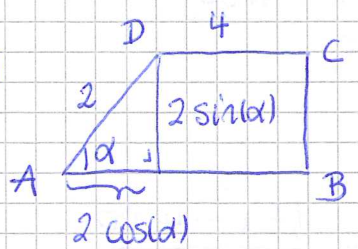
$$= \frac{10\,(v - 10\sqrt{57})(v + 10\sqrt{57})}{v^2}$$

v	$-\infty$ 0	$10\sqrt{57}$	$+\infty$
$P'(v)$	‖ $-$	0	$+$
$P(v)$	‖ $\searrow$	≈ 1510	$\nearrow$

min : $v = 10\sqrt{57} \cong 75,5$ km/h

⑦

a)



$$S(\alpha) = 4 \cdot 2 \cdot \sin(\alpha) + \frac{\cancel{2}\sin(\alpha) \cdot 2\cos(\alpha)}{\cancel{2}}$$

$$= 8\sin(\alpha) + 2\sin(\alpha)\cos(\alpha)$$

$\alpha \in \,]0; \frac{\pi}{2}[$

$$= 2\sin(\alpha)\,(4 + \cos(\alpha))$$

b) $$S'(\alpha) = 2\cos(\alpha)\,(4 + \cos(\alpha)) + 2\sin(\alpha)\,(-\sin(\alpha))$$

$$= 8\cos(\alpha) + 2\cos^2(\alpha) - 2\underbrace{\sin^2(\alpha)}_{1-\cos^2(\alpha)} = 4\cos^2(\alpha) + 8\cos(\alpha) - 2$$

$$= 2\,(2\cos^2(\alpha) + 4\cos(\alpha) - 1)$$

$$\Delta = 16 + 8 = 24 \Rightarrow \cos(\alpha) = \frac{-4 \pm 2\sqrt{6}}{4} = \frac{-2 \pm \sqrt{6}}{2}$$

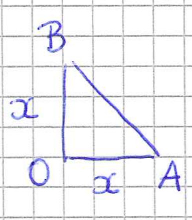
$$\cos(\alpha) = \frac{-2+\sqrt{6}}{2} \quad \Rightarrow \quad \alpha \cong 1,344 \qquad (-1,344 \text{ sol. à élim.})$$

$$\cos(\alpha) = \frac{-2-\sqrt{6}}{2} \quad \Rightarrow \quad \text{impossible car } -1 \leq \cos(\alpha) \leq 1$$

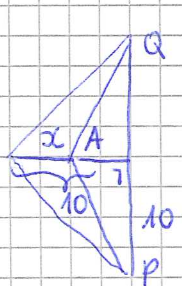
α	$-\infty$	0		$1,344$		$\pi/2$	$+\infty$
$S'(\alpha)$	//////	‖	$+$	0	$-$	‖	//////
$S(\alpha)$	//////	‖	$\nearrow$	$\sim 8,23$	$\searrow$	‖	//////

$$\Rightarrow \text{max} : \quad \alpha \cong 1,344 \quad \Rightarrow \quad \alpha \cong 77^\circ$$

(8)

a) 

Pythagore: $AB = \sqrt{x^2 + x^2} = \sqrt{2}\, x$



Pythagore: $AP = \sqrt{10^2 + (10-x)^2} = \sqrt{200 - 20x + x^2}$

hauteur de la pyramide:

Pythagore: $h = \sqrt{200 - 20x + x^2 - x^2} = \sqrt{200 - 20x} = \sqrt{20}\sqrt{10-x}$

b) $V(x) = 2x^2 \cdot \sqrt{20} \cdot \sqrt{10-x} \cdot \frac{1}{3} = \frac{2\sqrt{20}}{3}\, x^2 \sqrt{10-x}$

$x \in \,]0;10[$

c)
$$V'(x) = \frac{2\sqrt{20}}{3}\left[2x\sqrt{10-x} + x^2 \frac{(-1)}{2\sqrt{10-x}} \right]$$
$$= \frac{2\sqrt{20}}{3}\left[\frac{4x(10-x) - x^2}{2\sqrt{10-x}} \right] = \frac{2\sqrt{20}}{3} \cdot \frac{40x - 5x^2}{2\sqrt{10-x}}$$
$$= \frac{2\sqrt{20}}{3} \cdot \frac{5x(8-x)}{2\sqrt{10-x}}$$

x	$-\infty$	0		8		10	$+\infty$
$V'(x)$	//////	‖	$+$	0	$-$	‖	//////
$V(x)$	//////	‖	$\nearrow$	$\sim 269,85$	$\searrow$	‖	//////

$$\Rightarrow \text{max} : \quad x = 8 \text{ m}$$