Applications de la dérivée

Exercice 1.

a) $\lim\limits_{x\to 4} f(x) \overbrace{=}^{"\frac{0}{0}"\text{ (B-H)}} \lim\limits_{x\to 4} \dfrac{3}{3\,x^2-10} = \boxed{\dfrac{3}{38}}$

b) $\lim\limits_{x\to 0} f(x) \overbrace{=}^{"\frac{0}{0}"\text{ (B-H)}} \lim\limits_{x\to 0} \dfrac{\cos(x)-1}{6\,x}$

$\overbrace{=}^{"\frac{0}{0}"\text{ (B-H)}} \lim\limits_{x\to 0} \dfrac{-\sin(x)}{6} = \boxed{0}$

$\lim\limits_{x\to 5} f(x) \overbrace{=}^{"\frac{0}{0}"\text{ (B-H)}} \lim\limits_{x\to 5} \dfrac{3\,x^2-20}{1} = \boxed{55}$

$\lim\limits_{x\to 0} f(x) \overbrace{=}^{"\frac{0}{0}"\text{ (B-H)}} \lim\limits_{x\to 0} \dfrac{-\sin(x)}{14\,x}$

$\overbrace{=}^{"\frac{0}{0}"\text{ (B-H)}} \lim\limits_{x\to 0} \dfrac{-\cos(x)}{14} = \boxed{-\dfrac{1}{14}}$

Exercice 2.

a) $f(x) = 3x+5$
$g(x) = 2x^2-5x-5$

$f(-1) = 2 = g(-1) \quad \Rightarrow \boxed{I(-1;2)}$

b) $f'(x) = 3 \qquad g'(x) = 4x-5$

$m_1 = f'(-1) = 3 \qquad m_2 = g'(-1) = -9$

$\tan(\alpha) = \left|\dfrac{-9-3}{1-27}\right| = \dfrac{6}{13} \Rightarrow \boxed{\alpha \cong 24,78°}$

$f(x) = 4x-3$
$g(x) = 3x^2-2x-12$

$f(-1) = -7 = g(-1) \quad \Rightarrow \boxed{I(-1;-7)}$

$f'(x) = 4 \qquad g'(x) = 6x-2$

$,m_1 = f'(-1) = 4 \qquad m_2 = g'(-1) = -8$

$\tan(\alpha) = \left|\dfrac{-8-4}{1-32}\right| = \dfrac{12}{31} \Rightarrow \boxed{\alpha \cong 21,16°}$

Exercice 3.

$\boxed{ED(f) = \mathbb{R} - \{-1\}}$

$\Rightarrow$ zéro : $\boxed{x=3}$

x	$-\infty$		-1		3		$+\infty$
$f(x)$		$-$	‖	$+$	0	$+$	

$\boxed{ED(f) = \mathbb{R} - \{3\}}$

$\Rightarrow$ zéro : $\boxed{x=-1}$

x	$-\infty$		-1		3		$+\infty$
$f(x)$		$-$	0	$-$	‖	$+$	

$$\lim_{\substack{x\to -1\\<}} f(x) \overset{"\frac{16}{0_-}"}{\overbrace{=}} -\infty \text{ et } \lim_{\substack{x\to -1\\>}} f(x) \overset{"\frac{16}{0_+}"}{\overbrace{=}} +\infty$$

$\Rightarrow$ **AV : $x = -1$**

AO $\Rightarrow$ division polynomiale

$$\begin{array}{l} x^2 - 6x + 9 = (x+1)\,(x-7)\, + 16 \\ -x^2 \ \ - x \\ \hline \quad -7x + 9 \\ \quad\ \ 7x + 7 \\ \hline \qquad\quad 16 \end{array}$$

$\Rightarrow$ **AO : $y = x - 7$**

étude du signe de $\delta(x) = \dfrac{16}{x+1}$

x	$-\infty$	-1	$+\infty$
$\delta(x)$	$-$	||	$+$

f est dessus l'AO si $x \in]-1;+\infty[$
f est dessous l'AO si $x \in]-\infty;-1[$

$$f'(x) = \frac{(2x-6)\cdot(x+1) - (x^2-6x+9)\cdot 1}{(x+1)^2}$$
$$= \frac{x^2+2x-15}{(x+1)^2} = \frac{(x+5)(x-3)}{(x+1)^2}$$

$ED(f') = ED(f)$

x	$-\infty$		-5		-1		3		$+\infty$
$f'(x)$		$+$	0	$-$	||	$-$	0	$+$	
$f(x)$		↗	-16	↘	||	↘	0	↗	

$$\lim_{\substack{x\to 3\\<}} f(x) \overset{"\frac{16}{0_-}"}{\overbrace{=}} -\infty \text{ et } \lim_{\substack{x\to 3\\>}} f(x) \overset{"\frac{16}{0_+}"}{\overbrace{=}} +\infty$$

$\Rightarrow$ **AV : $x = 3$**

AO $\Rightarrow$ division polynomiale

$$\begin{array}{l} x^2 + 2x \ \ + 1 = (x-3)\,(x+5)\, + 16 \\ -x^2 + 3x \\ \hline \qquad 5x \ \ + 1 \\ \quad\ -5x + 15 \\ \hline \qquad\quad 16 \end{array}$$

$\Rightarrow$ **AO : $y = x + 5$**

étude du signe de $\delta(x) = \dfrac{16}{x-3}$

x	$-\infty$	3	$+\infty$
$\delta(x)$	$-$	||	$+$

f est dessus l'AO si $x \in]3;+\infty[$
f est dessous l'AO si $x \in]-\infty;3[$

$$f'(x) = \frac{(2x+2)\cdot(x-3) - (x^2+2x+1)}{(x-3)^2}$$
$$= \frac{x^2-6x-7}{(x-3)^2} = \frac{(x+1)(x-7)}{(x-3)^2}$$

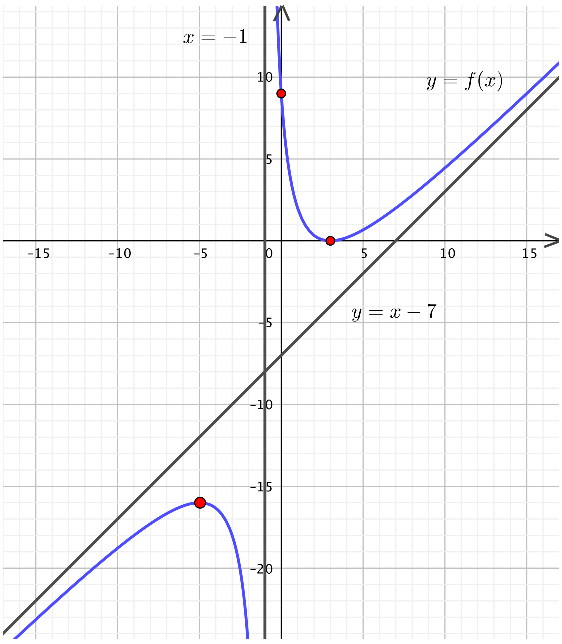
$ED(f') = ED(f)$

x	$-\infty$		-1		3		7		$+\infty$
$f'(x)$		$+$	0	$-$	||	$-$	0	$+$	
$f(x)$		↗	0	↘	||	↘	16	↗	

max : $(-5;-16)$

min : $(3;0)$

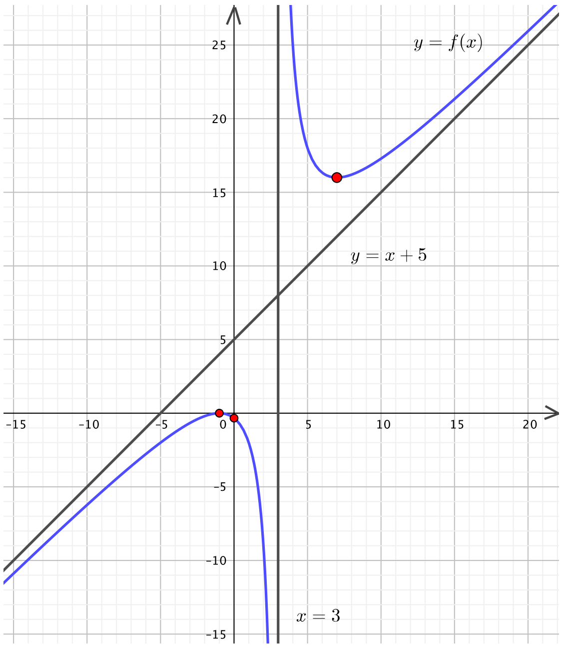
pt. part. : $(0;9)$



max : $(-1;0)$

min : $(7;16)$

pt. part. : $(0;-\frac{1}{3})$



Exercice 4.

a) $xy = 24 \quad \Leftrightarrow \quad y = \dfrac{24}{x}$

$A = (24-2x)(18-y) = 432-24y-36x+2xy$

$A(x) = 432-\dfrac{576}{x}-36x+48 = 480-\dfrac{576}{x}-36x$

$= \dfrac{-36x^2+480x-576}{x}$

$\boxed{= \dfrac{-12(3x^2-40x+48)}{x}}$

$xy = 12 \quad \Leftrightarrow \quad y = \dfrac{12}{x}$

$A = (30-2x)(20-y) = 600-30y-40x+2xy$

$A(x) = 600-\dfrac{360}{x}-40x+24 = 624-\dfrac{360}{x}-40x$

$= \dfrac{-40x^2+624x-360}{x}$

$\boxed{= \dfrac{-8(5x^2-78x+45)}{x}}$

b) $A'(x) = \dfrac{576}{x^2} - 36 = \dfrac{576 - 36x^2}{x^2}$

$= \dfrac{36(16 - x^2)}{x^2} = \dfrac{36(4 - x)(4 + x)}{x^2}$

$ED(A) = ED(A') =]0; 12]$

x	0		4		12
$A'(x)$		+	0	−	
$A(x)$		↗	192	↘	

L'aire est maximale si $x = 4$ m

c) $A(4) = 192 \text{ m}^2$

$A'(x) = \dfrac{360}{x^2} - 40 = \dfrac{360 - 40x^2}{x^2}$

$= \dfrac{40(9 - x^2)}{x^2} = \dfrac{40(3 - x)(3 + x)}{x^2}$

$ED(A) = ED(A') =]0; 15]$

x	0		3		15
$A'(x)$		+	0	−	
$A(x)$		↗	384	↘	

L'aire est maximale si $x = 3$ m

$A(3) = 384 \text{ m}^2$