Limites

Exercice 1.

a) $\dfrac{4x-1}{x+5} > 0$

x	$-\infty$	-5	$\frac{1}{4}$	$+\infty$
$\frac{4x-1}{x+5}$	$+$	‖	$-$ 0	$+$

$\Rightarrow$ $\boxed{ED(f) =]-\infty; -5[\cup]\frac{1}{4}; +\infty[}$

$\dfrac{3x-2}{x+8} > 0$

x	$-\infty$	-8	$\frac{2}{3}$	$+\infty$
$\frac{3x-2}{x+8}$	$+$	‖	$-$ 0	$+$

$\Rightarrow$ $\boxed{ED(f) =]-\infty; -8[\cup]\frac{2}{3}; +\infty[}$

b) zéro : $\dfrac{4x-1}{x+5} = 1 \quad \Rightarrow \quad 4x-1 = x+5$

$\Rightarrow \quad 3x = 6 \quad \Rightarrow \quad x = 2$

x	$-\infty$	-5	$\frac{1}{4}$	2	$+\infty$
$f(x)$	$+$	(grisé)	$-$	0	$+$

zéro : $\dfrac{3x-2}{x+8} = 1 \quad \Rightarrow \quad 3x-2 = x+8$

$\Rightarrow \quad 2x = 10 \quad \Rightarrow \quad x = 5$

x	$-\infty$	-8	$\frac{2}{3}$	5	$+\infty$
$f(x)$	$+$	(grisé)	$-$	0	$+$

Exercice 2.

a) $\lim\limits_{x\to-\infty} \dfrac{3x^2-5x}{x-4} = \lim\limits_{x\to-\infty} \dfrac{3x^2}{x}$

$= \lim\limits_{x\to-\infty} 3x = \boxed{-\infty}$

$\lim\limits_{x\to-\infty} \dfrac{4x^2-5x}{x-3} = \lim\limits_{x\to-\infty} \dfrac{4x^2}{x}$

$= \lim\limits_{x\to-\infty} 4x = \boxed{-\infty}$

b) $\lim\limits_{\substack{x\to-3\\<}} \dfrac{-4x}{x+3} \overset{\text{"}\frac{12}{0_-}\text{"}}{\overbrace{=}} \boxed{-\infty}$

$\lim\limits_{\substack{x\to-6\\>}} \dfrac{7x}{x+6} \overset{\text{"}\frac{-42}{0_+}\text{"}}{\overbrace{=}} \boxed{-\infty}$

c) $\lim\limits_{x\to+\infty} \dfrac{x^4-7x^5+3x^2}{2x^6-4} = \lim\limits_{x\to+-\infty} \dfrac{-7x^5}{2x^6}$

$= \lim\limits_{x\to+\infty} \dfrac{-7}{2x} = \boxed{0}$

$\lim\limits_{x\to+\infty} \dfrac{2x^6-4}{x^4-7x^7+3x^2} = \lim\limits_{x\to+-\infty} \dfrac{2x^6}{-7x^7}$

$= \lim\limits_{x\to+\infty} \dfrac{2}{-7x} = \boxed{0}$

d) $\lim_{x\to -3} \frac{x^2+4x+3}{x+3} \overset{\text{"}\frac{0}{0}\text{" f.i.}}{\overbrace{=}} \lim_{x\to -3} \frac{(x+3)(x+1)}{x+3}$

$= \lim_{x\to -3} x+1 = \boxed{-2}$

$\lim_{x\to 5} \frac{x^2-6x+5}{x-5} \overset{\text{"}\frac{0}{0}\text{" f.i.}}{\overbrace{=}} \lim_{x\to 5} \frac{(x-1)(x-5)}{x-5}$

$= \lim_{x\to 5} x-1 = \boxed{4}$

e) $\lim_{x\to 2} \frac{\sqrt{x+14}-4}{x-2} \overset{\text{"}\frac{0}{0}\text{" f.i.}}{\overbrace{=}}$

$\lim_{x\to 2} \frac{x-2}{(x-2)(\sqrt{x+14}+4)} =$

$\lim_{x\to 2} \frac{1}{\sqrt{x+14}+4} = \boxed{\frac{1}{8}}$

$\lim_{x\to 6} \frac{\sqrt{x+19}-5}{x-6} \overset{\text{"}\frac{0}{0}\text{" f.i.}}{\overbrace{=}}$

$\lim_{x\to 6} \frac{x-6}{(x-6)(\sqrt{x+19}+5)} =$

$\lim_{x\to 6} \frac{1}{\sqrt{x+19}+5} = \boxed{\frac{1}{10}}$

f) $\lim_{x\to -\infty} \frac{\sqrt{16x^2}}{7-2x} = \lim_{x\to -\infty} \frac{4\overbrace{|x|}^{-x}}{-2x} = \boxed{2}$

$\lim_{x\to -\infty} \frac{\sqrt{9x^2}}{4-6x} = \lim_{x\to -\infty} \frac{3\overbrace{|x|}^{-x}}{-6x} = \boxed{\frac{1}{2}}$

Exercice 3.

a) $f(x) = \frac{x^2(x+2)}{(x+1)(x-3)}$

$\boxed{ED(f) = \mathbb{R} - \{-1;3\}}$

b) zéros : $x=-2$, $x=0$

x	$-\infty$	-2		-1		0		3	$+\infty$
$f(x)$	$-$	0	$+$	‖	$-$	0	$-$	‖	$+$

c) $\lim_{\substack{x\to -1\\<}} f(x) \overset{\text{"}\frac{1}{0_+}\text{"}}{\overbrace{=}} +\infty$ et $\lim_{\substack{x\to -1\\>}} f(x) \overset{\text{"}\frac{1}{0_-}\text{"}}{\overbrace{=}} -\infty$

$\lim_{\substack{x\to 3\\<}} f(x) \overset{\text{"}\frac{45}{0_-}\text{"}}{\overbrace{=}} -\infty$ et $\lim_{\substack{x\to 3\\>}} f(x) \overset{\text{"}\frac{45}{0_+}\text{"}}{\overbrace{=}} +\infty$

$\Rightarrow \boxed{\text{AV : } x=-1 \text{ et } x=3}$

$f(x) = \frac{x^2(x+4)}{(x+2)(x-3)}$

$\boxed{ED(f) = \mathbb{R} - \{-2;3\}}$

zéros : $x=-4$, $x=0$

x	$-\infty$	-4		-2		0		3	$+\infty$
$f(x)$	$-$	0	$+$	‖	$-$	0	$-$	‖	$+$

$\lim_{\substack{x\to -2\\<}} f(x) \overset{\text{"}\frac{8}{0_+}\text{"}}{\overbrace{=}} +\infty$ et $\lim_{\substack{x\to -2\\>}} f(x) \overset{\text{"}\frac{8}{0_-}\text{"}}{\overbrace{=}} -\infty$

$\lim_{\substack{x\to 3\\<}} f(x) \overset{\text{"}\frac{63}{0_-}\text{"}}{\overbrace{=}} -\infty$ et $\lim_{\substack{x\to 3\\>}} f(x) \overset{\text{"}\frac{63}{0_+}\text{"}}{\overbrace{=}} +\infty$

$\Rightarrow \boxed{\text{AV : } x=-2 \text{ et } x=3}$

$\lim_{x\to\pm\infty} f(x) = \pm\infty \quad \Rightarrow$ pas d'AH

AO $\Rightarrow$ division polynomiale

$$\begin{array}{l} x^3 + 2x^2 = \left(x^2 - 2x - 3\right)\left(x+4\right) + 11x + 12 \\ -\ x^3 + 2x^2 + 3x \\ \hline \quad 4x^2 + 3x \\ \quad -\ 4x^2 + 8x + 12 \\ \hline \quad\quad 11x + 12 \end{array}$$

$\Rightarrow$ $\boxed{\text{AO} : y = x + 4}$

étude du signe de $\delta(x) = \dfrac{11x+12}{x^2-2x-3}$

x	$-\infty$	$-\frac{12}{11}$		-1		3		$+\infty$
$\delta(x)$	$-$	0	$+$	‖	$-$	‖	$+$	

f est dessus l'AO si $x \in]-\frac{12}{11}; -1[\cup]3; +\infty[$

f est dessous l'AO si $x \in]-\infty; -\frac{12}{11}[\cup]-1; 3[$

la courbe coupe l'AO en $\left(-\frac{12}{11}; \frac{36}{11}\right)$

$\lim_{x\to\pm\infty} f(x) = \pm\infty \quad \Rightarrow$ pas d'AH

AO $\Rightarrow$ division polynomiale

$$\begin{array}{l} x^3 + 4x^2 = \left(x^2 - x - 6\right)\left(x+5\right) + 11x + 30 \\ -\ x^3 \ + x^2 + 6x \\ \hline \quad 5x^2 + 6x \\ \quad -\ 5x^2 + 5x + 30 \\ \hline \quad\quad 11x + 30 \end{array}$$

$\Rightarrow$ $\boxed{\text{AO} : y = x + 5}$

étude du signe de $\delta(x) = \dfrac{11x+30}{x^2-x-6}$

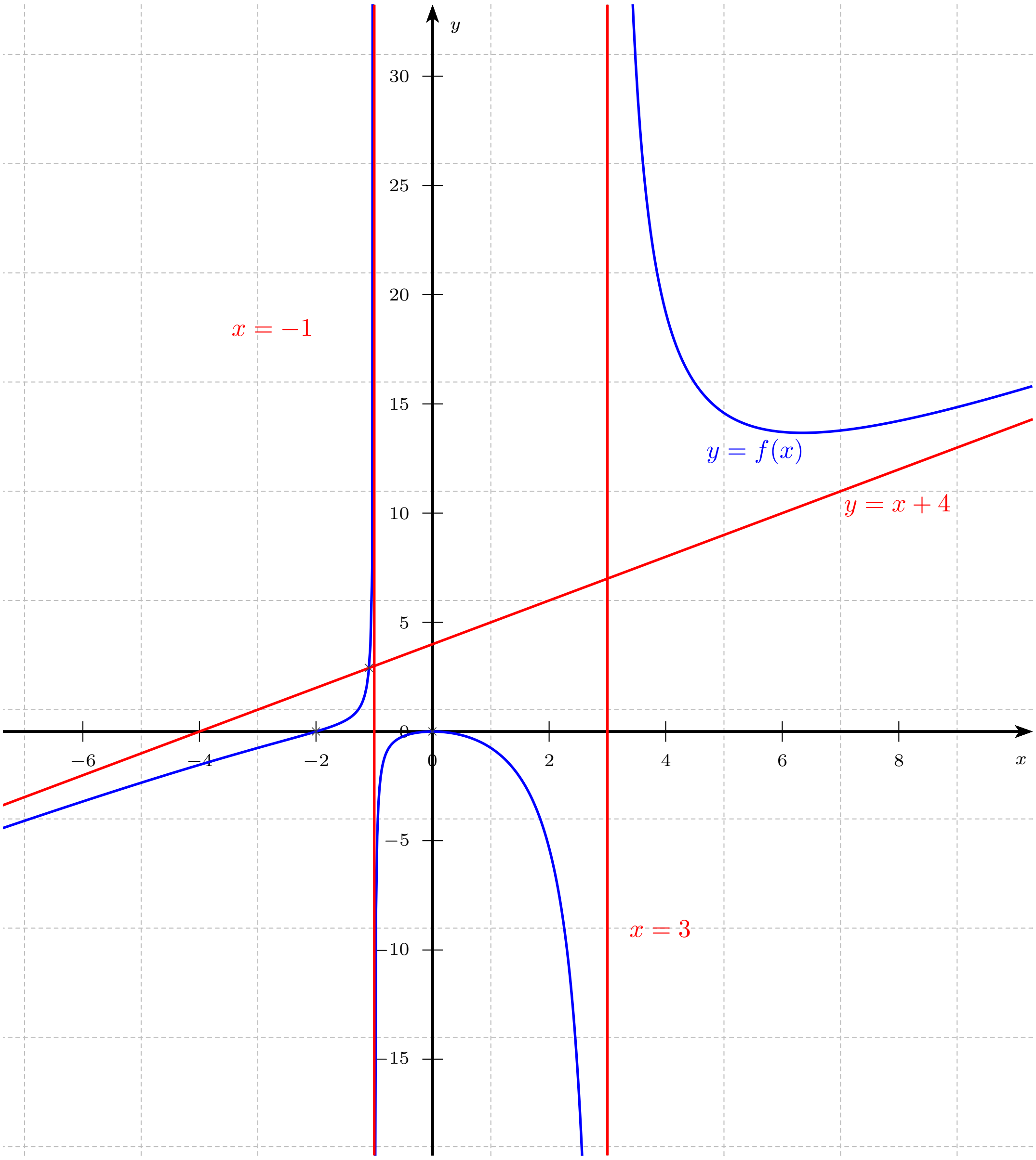
x	$-\infty$	$-\frac{30}{11}$		-2		3		$+\infty$
$\cdots$	$-$	0	$+$	‖	$-$	‖	$+$	

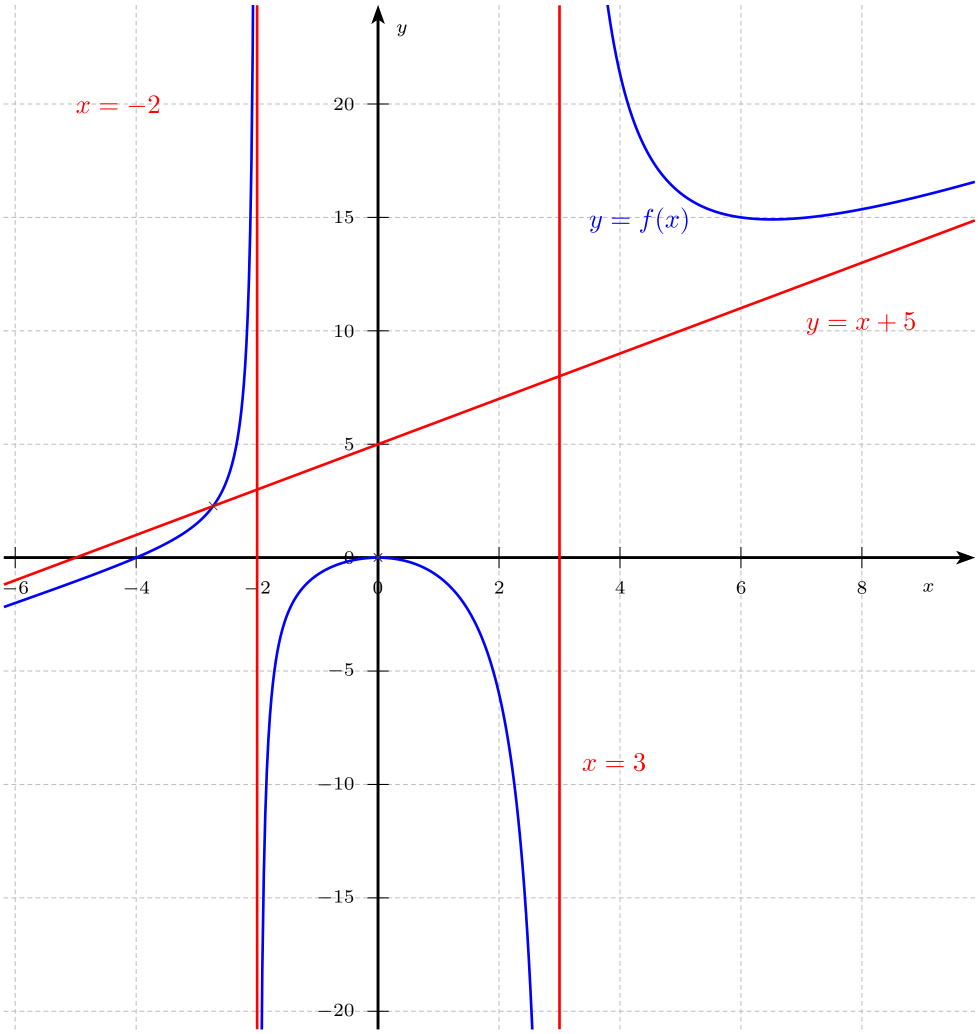
f est dessus l'AO si $x \in]-\frac{30}{11}; -2[\cup]3; +\infty[$

f est dessous l'AO si $x \in]-\infty; -\frac{30}{11}[\cup]-2; 3[$

la courbe coupe l'AO en $\left(-\frac{30}{11}; \frac{25}{11}\right)$

d)





$x = -2$
$y = f(x)$
$y = x + 5$
$x = 3$
y
x
20
15
10
5
0
−5
−10
−15
−20
−6
−4
−2
2
4
6
8