

## Puissances et racines

### Exercice 1.

$$a) a^{3-\frac{2}{3}} = a^{\frac{7}{3}} = \sqrt[3]{a^7} = a^2 \cdot \sqrt[3]{a}$$

$$b^{4-\frac{2}{5}} = b^{\frac{18}{5}} = \sqrt[5]{b^{18}} = b^3 \cdot \sqrt[5]{b^3}$$

$$b) b^{\frac{3}{4}+\frac{5}{8}} = b^{\frac{11}{8}} = \sqrt[8]{b^{11}} = b \cdot \sqrt[8]{b^3}$$

$$a^{\frac{5}{3}+\frac{7}{6}} = a^{\frac{17}{6}} = \sqrt[6]{a^{17}} = a^2 \cdot \sqrt[6]{a^5}$$

$$c) 5^{\frac{12}{5}-3} = 5^{-\frac{3}{5}} = \frac{1}{\sqrt[5]{5^3}}$$

$$3^{\frac{2}{3}-\frac{3}{4}} = 3^{-\frac{1}{12}} = \frac{1}{\sqrt[12]{3}}$$

### Exercice 2.

$$a) a^{\frac{1}{5}-\frac{1}{2}} = a^{-\frac{3}{10}} = \frac{1}{a^{\frac{3}{10}}}$$

$$a^{\frac{1}{4}-\frac{1}{2}} = a^{-\frac{1}{4}} = \frac{1}{a^{\frac{1}{4}}}$$

$$b) b^{\frac{9}{2}-4-\frac{2}{5}} = b^{\frac{1}{10}}$$

$$b^{\frac{15}{2}-6-\frac{3}{5}} = b^{\frac{9}{10}}$$

$$c) c^{\frac{1}{5}-\frac{1}{6}} = c^{\frac{1}{30}}$$

$$c^{\frac{1}{10}-\frac{1}{12}} = c^{\frac{1}{60}}$$

$$d) \sqrt{11 \sqrt[3]{11^{1+\frac{1}{2}}}} = \sqrt{11 \cdot \left(11^{\frac{3}{2}}\right)^{\frac{1}{3}}} = \sqrt{11^{1+\frac{1}{2}}}$$

$$= \left(11^{\frac{3}{2}}\right)^{\frac{1}{2}} = 11^{\frac{3}{4}}$$

$$\sqrt{7 \sqrt[4]{7^{1+\frac{1}{2}}}} = \sqrt{7 \cdot \left(7^{\frac{3}{2}}\right)^{\frac{1}{4}}} = \sqrt{7^{1+\frac{3}{8}}}$$

$$= \left(7^{\frac{11}{8}}\right)^{\frac{1}{2}} = 7^{\frac{11}{16}}$$

$$e) a^{\frac{3}{10}-\frac{1}{6}} = a^{\frac{4}{30}} = a^{\frac{2}{15}}$$

$$a^{\frac{2}{3}-\frac{5}{8}} = a^{\frac{1}{24}}$$

$$f) \left(\frac{a^9}{b^3}\right)^{\frac{4}{3}} = \frac{a^{12}}{b^4}$$

$$\left(\frac{b^3}{a^6}\right)^{\frac{2}{3}} = \frac{b^2}{a^4}$$

**Exercice 3.**

a)  $x^{\frac{1}{2}} = 25 \Rightarrow x = 25^2 = \boxed{625}$

$x^{\frac{1}{3}} = 27 \Rightarrow x = 27^3 = \boxed{19683}$

b)  $x^3 = 125 \Rightarrow x = \sqrt[3]{125} = \boxed{5}$

$x^4 = 16 \Rightarrow x = \pm\sqrt[4]{16} = \boxed{\pm 2}$

c)  $x^2 - 2x = 0 \Rightarrow x(x - 2) = 0$

$\Rightarrow \mathcal{S} = \{0; 2\}$

$x^2 - 3x = 0 \Rightarrow x(x - 3) = 0$

$\Rightarrow \mathcal{S} = \{0; 3\}$