Exponentielles et logarithmes

Exercice 1.

a) $7^{2x-1} = 2401$	$4^{5x-1} = 16384$
$\Rightarrow \quad 2x - 1 = \log_7(2401) = 4$	$\Rightarrow \quad 5x - 1 = \log_4(16384) = 7$
$\Rightarrow \quad 2x = 5 \quad \Rightarrow \quad x = \frac{5}{2}$	$\Rightarrow \quad 5x = 8 \quad \Rightarrow \quad x = \frac{8}{5}$
$\Rightarrow \boxed{S = \left\{\frac{5}{2}\right\}}$	$\Rightarrow \boxed{S = \left\{\frac{8}{5}\right\}}$
b) $ED =]2; +\infty[$	$ED =]-\infty; 0[$
$\log_8(x^2 - 4) = \log_8(8) + \log_8(x + 2)$	$\log_8(x^2 - 6x) = \log_8(8) + \log_8(3 - x)$
$\Rightarrow \quad \log_8(x^2 - 4) = \log_8(8(x + 2))$	$\Rightarrow \quad \log_8(x^2 - 6x) = \log_8(8(3 - x))$
$\Rightarrow \quad x^2 - 4 = 8x + 16 \quad \Rightarrow \quad x^2 - 8x - 20 = 0$	$\Rightarrow \quad x^2 - 6x = 24 - 8x \Rightarrow x^2 + 2x - 24 = 0$
$\Rightarrow \quad (x - 10)(x + 2) = 0$	$\Rightarrow \quad (x + 6)(x - 4) = 0$
$\Rightarrow \quad x = 10$ et $x = -2$ (sol. à élim.)	$\Rightarrow \quad x = -6$ et $x = 4$ (sol. à élim.)
$\Rightarrow \boxed{S = \{10\}}$	$\Rightarrow \boxed{S = \{-6\}}$
c) $x^2 = 196 \quad \Rightarrow \quad x = \sqrt{196} = 14$	$x^2 = 169 \quad \Rightarrow \quad x = \sqrt{169} = 13$
$\Rightarrow \boxed{S = 14}$	$\Rightarrow \boxed{S = 13}$

Exercice 2.

a) $N(20) = \dfrac{200}{1+300\cdot e^{-3,2}} \simeq 15.12$

$\Rightarrow$ $\boxed{\text{15 chimpanzés infectés}}$

$N(15) = \dfrac{300}{1+400\cdot e^{-2,7}} \simeq 10.76$

$\Rightarrow$ $\boxed{\text{11 chimpanzés infectés}}$

b) $\dfrac{200}{1+300\cdot e^{-0,16t}} = 120$

$\Rightarrow \quad 200 = 120\left(1+300\cdot e^{-0,16t}\right)$

$\Rightarrow \quad 1+300\cdot e^{-0,16t} = \dfrac{5}{3}$

$\Rightarrow \quad 300\cdot e^{-0,16t} = \dfrac{2}{3} \quad \Rightarrow \quad e^{-0,16t} = \dfrac{2}{900}$

$\Rightarrow \quad -0,16t = \ln\left(\frac{2}{900}\right)$

$\Rightarrow \quad t = -\frac{1}{0,16}\cdot\ln\left(\frac{2}{900}\right) \simeq 38.18$

$\Rightarrow$ $\boxed{\text{dans 39 jours}}$

$\dfrac{300}{1+400\cdot e^{-0,18t}} = 160$

$\Rightarrow \quad 300 = 160\left(1+400\cdot e^{-0,18t}\right)$

$\Rightarrow \quad 1+400\cdot e^{-0,18t} = \dfrac{15}{8}$

$\Rightarrow \quad 400\cdot e^{-0,18t} = \dfrac{7}{8} \quad \Rightarrow \quad e^{-0,18t} = \dfrac{7}{3200}$

$\Rightarrow \quad -0,18t = \ln\left(\frac{7}{3200}\right)$

$\Rightarrow \quad t = -\frac{1}{0,18}\cdot\ln\left(\frac{7}{3200}\right) \simeq 34.03$

$\Rightarrow$ $\boxed{\text{dans 35 jours}}$

Exercice 3.

a) $\dfrac{4x-1}{x+5} > 0$

x	$-\infty$	-5		$\frac{1}{4}$		$+\infty$
$\frac{4x-1}{x+5}$	$+$	‖	$-$	0	$+$	

$\Rightarrow$ $\boxed{ED(f) =]-\infty;-5[\cup]\tfrac{1}{4};+\infty[}$

$\dfrac{3x-2}{x+8} > 0$

x	$-\infty$	-8		$\frac{2}{3}$		$+\infty$
$\frac{3x-2}{x+8}$	$+$	‖	$-$	0	$+$	

$\Rightarrow$ $\boxed{ED(f) =]-\infty;-8[\cup]\tfrac{2}{3};+\infty[}$

b) zéro : $\dfrac{4x-1}{x+5} = 1 \quad \Rightarrow \quad 4x-1 = x+5$

$\Rightarrow \quad 3x = 6 \quad \Rightarrow \quad x = 2$

x	$-\infty$	-5		$\frac{1}{4}$		2		$+\infty$
$f(x)$	$+$	‖	(zone grisée)	‖	$-$	0	$+$	

zéro : $\dfrac{3x-2}{x+8} = 1 \quad \Rightarrow \quad 3x-2 = x+8$

$\Rightarrow \quad 2x = 10 \quad \Rightarrow \quad x = 5$

x	$-\infty$	-8		$\frac{2}{3}$		5		$+\infty$
$f(x)$	$+$	‖	(zone grisée)	‖	$-$	0	$+$	

Exercice 4.

a) $\boxed{Q(t) = 76 \cdot 0,5^{t/4}}$	$\boxed{Q(t) = 84 \cdot 0,5^{t/5}}$
b) $Q(2) = 76 \cdot 0,5^{1/2} \simeq \boxed{53.74 \text{ mg}}$	$Q(3) = 84 \cdot 0,5^{3/5} \simeq \boxed{55.42 \text{ mg}}$
c) $20 = 76 \cdot 0,5^{t/4} \quad \Rightarrow \quad 0,5^{t/4} = \dfrac{5}{19}$ $\Rightarrow \quad \dfrac{t}{4} = \log_{0,5}\left(\frac{5}{19}\right)$ $\Rightarrow \quad t = 4 \cdot \log_{0,5}\left(\frac{5}{19}\right) = 7.704$ $\Rightarrow$ $\boxed{\text{Après 7 h 42 min et 15 s}}$	$30 = 84 \cdot 0,5^{t/5} \quad \Rightarrow \quad 0,5^{t/5} = \dfrac{15}{42}$ $\Rightarrow \quad \dfrac{t}{5} = \log_{0,5}\left(\frac{15}{42}\right)$ $\Rightarrow \quad t = 5 \cdot \log_{0,5}\left(\frac{15}{42}\right) = 7.427$ $\Rightarrow$ $\boxed{\text{Après 7 h 25 min et 38 s}}$

Exercice 5.

a) $C_{15} = 30'000 \cdot 1,04^{15} \simeq 54028.31$ $\Rightarrow$ $\boxed{\text{54'028.30 fr.}}$	$C_{10} = 25'000 \cdot 1,03^{10} \simeq 33597.91$ $\Rightarrow$ $\boxed{\text{33'597.90 fr.}}$
b) $72'000 = 30'000 \cdot 1,04^n \quad \Rightarrow \quad 1,04^n = \dfrac{12}{5}$ $\Rightarrow \quad n = \log_{1,04}\left(\frac{12}{5}\right) \simeq 22.3$ $\Rightarrow$ $\boxed{\text{Après 8 (23 – 15) années}}$	$65'000 = 25'000 \cdot 1,03^n \quad \Rightarrow \quad 1,03^n = \dfrac{13}{5}$ $\Rightarrow \quad n = \log_{1,03}\left(\frac{13}{5}\right) \simeq 32.3$ $\Rightarrow$ $\boxed{\text{Après 23 (33 – 10) années}}$
c) $72'000 = 30'000 \cdot \left(1 + \frac{t}{100}\right)^{15}$ $\Rightarrow \quad \left(1 + \frac{t}{100}\right)^{15} = \dfrac{12}{5}$ $\Rightarrow \quad 1 + \dfrac{t}{100} = \sqrt[15]{\dfrac{12}{5}}$ $\Rightarrow \dfrac{t}{100} = \sqrt[15]{\dfrac{12}{5}} - 1 \simeq 0.0601$ $\Rightarrow$ $\boxed{\text{Un taux d'environ 6\%}}$	$65'000 = 25'000 \cdot \left(1 + \frac{t}{100}\right)^{10}$ $\Rightarrow \quad \left(1 + \frac{t}{100}\right)^{10} = \dfrac{13}{5}$ $\Rightarrow \quad 1 + \dfrac{t}{100} = \sqrt[10]{\dfrac{13}{5}}$ $\Rightarrow \dfrac{t}{100} = \sqrt[10]{\dfrac{13}{5}} - 1 \simeq 0.100$ $\Rightarrow$ $\boxed{\text{Un taux d'environ 10\%}}$

BONUS

$\log(8^{3140}) = 3140 \cdot \log(8) \simeq 2835.70$

$\log(9^{3274}) = 3274 \cdot \log(9) \simeq 3124.2$

$\Rightarrow \quad \boxed{b > a}$

$\Rightarrow \quad \boxed{b > a}$