

Fonctions III

Exercice 1.

$$a) f'(x) = 30x^4 - \frac{21}{4}x^2 - 6x + 7$$

$$b) f'(x) = 35(5x - 1)^6$$

$$c) f'(x) = 6(2x + 5)^2(-6x + 1)^4 - (2x + 5)^3 24(-6x + 1)^3 =$$

$$6(2x + 5)^2(-6x + 1)^3[(-6x + 1) - 4(2x + 5)]$$

$$= 6(2x + 5)^2(-6x + 1)^3(-14x - 19)$$

$$d) f'(x) = \frac{4(x^2 + 1) - (4x - 1)2x}{(x^2 + 1)^2}$$

$$= -\frac{2(2x^2 - x - 2)}{(x^2 + 1)^2}$$

$$e) f'(x) = \frac{5x}{\sqrt{5x^2 + 3}}$$

$$f) f'(x) = \frac{5}{3}(x^2 - x - 1)^{\frac{2}{3}}(2x - 1)$$

$$= \frac{5}{3}\sqrt[3]{(x^2 - x - 1)^2(2x - 1)}$$

$$f'(x) = 48x^5 - 12x^3 + \frac{6}{7}x - 9$$

$$f'(x) = 12(2x - 9)^5$$

$$f'(x) = 15(3x - 7)^4(-4x + 3)^3 - (3x - 7)^5 12(-4x + 3)^2 =$$

$$3(3x - 7)^4(-4x + 3)^2[5(-4x + 3) - 4(3x - 7)]$$

$$= 3(3x - 7)^4(-4x + 3)^2(-32x + 43)$$

$$f'(x) = \frac{2x(3x - 1) - (x^2 - 3)3}{(3x - 1)^2}$$

$$= \frac{3x^2 - 2x + 9}{(3x - 1)^2}$$

$$f'(x) = \frac{6x^3}{\sqrt{3x^4 + 2}}$$

$$f'(x) = \frac{7}{4}(x^2 + x + 1)^{\frac{3}{4}}(2x + 1)$$

$$= \frac{7}{4}\sqrt[4]{(x^2 + x + 1)^3(2x + 1)}$$

Exercice 2.

$$f'(x) = \frac{3(x+4) - (3x-2) \cdot 1}{(x+4)^2} = \frac{14}{(x+4)^2}$$

$$f(-2) = -4 \quad f'(-2) = \frac{7}{2}$$

$$(t) : y + 4 = \frac{7}{2}(x + 2)$$

$$\Rightarrow (t) : y = \frac{7}{2}x + 3$$

$$f'(x) = \frac{7(x+5) - (7x-1) \cdot 1}{(x+5)^2} = \frac{36}{(x+5)^2}$$

$$f(-1) = -2 \quad f'(-1) = \frac{9}{4}$$

$$(t) : y + 2 = \frac{9}{4}(x + 1)$$

$$\Rightarrow (t) : y = \frac{9}{4}x + \frac{1}{4}$$

Exercice 3.

$$a) f(3) = \frac{1}{3} \Rightarrow T\left(3; \frac{1}{3}\right)$$

$$f'(x) = -\frac{1}{x^2} \Rightarrow f'(3) = -\frac{1}{9}$$

$$(t) : y - \frac{1}{3} = -\frac{1}{9}(x - 3)$$

$$\Rightarrow (t) : y = -\frac{1}{9}x + \frac{2}{3}$$

$$b) y = 0 \Rightarrow x = 6 \Rightarrow A(6; 0)$$

$$x = 0 \Rightarrow y = \frac{2}{3} \Rightarrow B\left(0; \frac{2}{3}\right)$$

c) M milieu de AB

$$\Rightarrow M\left(\frac{6+0}{2}; \frac{0+\frac{2}{3}}{2}\right) \Rightarrow M\left(3; \frac{1}{3}\right)$$

$$\Rightarrow M = T$$

$$f(5) = \frac{1}{5} \Rightarrow T\left(5; \frac{1}{5}\right)$$

$$f'(x) = -\frac{1}{x^2} \Rightarrow f'(5) = -\frac{1}{25}$$

$$(t) : y - \frac{1}{5} = -\frac{1}{25}(x - 5)$$

$$\Rightarrow (t) : y = -\frac{1}{25}x + \frac{2}{5}$$

$$y = 0 \Rightarrow x = 10 \Rightarrow A(10; 0)$$

$$x = 0 \Rightarrow y = \frac{2}{5} \Rightarrow B\left(0; \frac{2}{5}\right)$$

M milieu de AB

$$\Rightarrow M\left(\frac{10+0}{2}; \frac{0+\frac{2}{5}}{2}\right) \Rightarrow M\left(5; \frac{1}{5}\right)$$

$$\Rightarrow M = T$$

BONUS

$$-3x^2 \cos(\cos(x^3)) \sin(x^3)$$

$$-4x^3 \cos(x^4) \sin(\sin(x^4))$$