Fonctions II

Exercice 1.

a) $\lim\limits_{x\to 3} \dfrac{2x-6}{x^2+x-12} \Rightarrow$ f.i. ("$\frac{0}{0}$")

$\lim\limits_{x\to 3} \dfrac{2(x-3)}{(x+4)(x-3)} = \lim\limits_{x\to 2} \dfrac{2}{x+4} = \boxed{\dfrac{2}{7}}$

$\lim\limits_{x\to 4} \dfrac{x^2+x-20}{x-4} \Rightarrow$ f.i. ("$\frac{0}{0}$")

$\lim\limits_{x\to 4} \dfrac{(x+5)(x-4)}{x-4} = \lim\limits_{x\to 4} (x+5) = \boxed{9}$

b) $\lim\limits_{\substack{x\to -5\\ >}} \dfrac{-2x}{x+5} \overbrace{=}^{"\frac{10}{0_+}"} \boxed{+\infty}$

$\lim\limits_{\substack{x\to -8\\ >}} \dfrac{6x}{x+8} \overbrace{=}^{"\frac{-48}{0_+}"} \boxed{-\infty}$

c) $\lim\limits_{x\to -\infty} \dfrac{-6x^5}{7x^5} = \boxed{-\dfrac{6}{7}}$

$\lim\limits_{x\to -\infty} \dfrac{2x^6}{-9x^6} = \boxed{-\dfrac{2}{9}}$

d) $\lim\limits_{x\to 1} \dfrac{x^3-5x+4}{x-1} \Rightarrow$ f.i. ("$\frac{0}{0}$")

$\lim\limits_{x\to 1} \dfrac{(x^2+x-4)(x-1)}{x-1} =$

$\lim\limits_{x\to 1} (x^2+x-4) = \boxed{-2}$

$\lim\limits_{x\to 1} \dfrac{x^3-7x^2+6}{x-1} \Rightarrow$ f.i. ("$\frac{0}{0}$")

$\lim\limits_{x\to 1} \dfrac{(x^2-6\,x-6)(x-1)}{x-1} =$

$\lim\limits_{x\to 1} (x^2-6x-6) = \boxed{-11}$

e) $\lim\limits_{x\to -2} \dfrac{\sqrt{x+18}-4}{x+2} \Rightarrow$ f.i. ("$\frac{0}{0}$")

$\lim\limits_{x\to -2} \dfrac{x+2}{(x+2)(\sqrt{x+18}+4)} =$

$\lim\limits_{x\to -2} \dfrac{1}{\sqrt{x+18}+4} = \boxed{\dfrac{1}{8}}$

$\lim\limits_{x\to -5} \dfrac{\sqrt{x+14}-3}{x+5} \Rightarrow$ f.i. ("$\frac{0}{0}$")

$\lim\limits_{x\to -5} \dfrac{x+5}{(x+5)(\sqrt{x+14}+3)} =$

$\lim\limits_{x\to -5} \dfrac{1}{\sqrt{x+14}+3} = \boxed{\dfrac{1}{6}}$

f) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{16x^2}}{7-5x} = \lim\limits_{x\to -\infty} \dfrac{4\overbrace{|x|}^{-x}}{-5x} = \boxed{\dfrac{4}{5}}$

$\lim\limits_{x\to -\infty} \dfrac{\sqrt{9x^2}}{4-7x} = \lim\limits_{x\to -\infty} \dfrac{3\overbrace{|x|}^{-x}}{-7x} = \boxed{\dfrac{3}{7}}$

Exercice 2.

a) AV : $x=-1$ et $x=4$ AH : $y=-2$

b) $f(x)=\dfrac{P(x)}{Q(x)}$

$Q(x)=(x+1)(x-4)=x^2-3x-4$

$P(x)=-2x(x-3)=-2x^2+6x$

$\Rightarrow \boxed{f(x)=\dfrac{-2x^2+6x}{x^2-3x-4}}$

AV : $x=-1$ et $x=3$ AH : $y=2$

$f(x)=\dfrac{P(x)}{Q(x)}$

$Q(x)=(x+1)(x-3)=x^2-2x-3$

$P(x)=2x(x-2)=2x^2-4x$

$\boxed{f(x)=\dfrac{2x^2-4x}{x^2-2x-3}}$

Exercice 3.

a) $f(x)=\dfrac{x^3-2x^2-2x-3}{(x+1)(x-3)}$

$\Rightarrow \boxed{ED(f)=\mathbb{R}-\{-1;3\}}$

b) $\lim\limits_{\substack{x\to-1\\<}} f(x) \overset{"\frac{-4}{0_+}"}{\overbrace{=}} -\infty$ et $\lim\limits_{\substack{x\to-1\\>}} f(x) \overset{"\frac{-4}{0_-}"}{\overbrace{=}} +\infty$

$\Rightarrow$ AV : $\boxed{x=-1}$

$\lim\limits_{x\to3} f(x) \quad \Rightarrow$ f.i. ("$\frac{0}{0}$")

$\lim\limits_{x\to3}\dfrac{(x^2+x+1)(x-3)}{(x+1)(x-3)}=$

$\lim\limits_{x\to3}\dfrac{x^2+x+1}{x+1}=\dfrac{13}{4}\Rightarrow$ trou $\left(3;\dfrac{13}{4}\right)$

AO $\Rightarrow$ division polynomiale

$$\begin{array}{l} x^3-2x^2-2x-3=\left(x^2-2x-3\right)x+x-3 \\ -x^3+2x^2+3x \\ \hline \qquad x-3 \end{array}$$

$f(x)=\dfrac{x^3-x^2-x-2}{(x+2)(x-2)}$

$\boxed{ED(f)=\mathbb{R}-\{-2;2\}}$

$\lim\limits_{\substack{x\to-2\\<}} f(x) \overset{"\frac{-12}{0_+}"}{\overbrace{=}} -\infty$ et $\lim\limits_{\substack{x\to-2\\>}} f(x) \overset{"\frac{-12}{0_-}"}{\overbrace{=}} +\infty$

$\Rightarrow$ AV : $\boxed{x=-2}$

$\lim\limits_{x\to2} f(x) \quad \Rightarrow$ f.i. ("$\frac{0}{0}$")

$\lim\limits_{x\to2}\dfrac{(x^2+x+1)(x-2)}{(x+2)(x-2)}=$

$\lim\limits_{x\to2}\dfrac{x^2+x+1}{x+2}=\dfrac{7}{4}\Rightarrow$ trou $\left(2;\dfrac{7}{4}\right)$

AO $\Rightarrow$ division polynomiale

$$\begin{array}{rrrrl}
x^3 & -x^2 & -x & -2 & = \left(x^2-4\right)\left(x-1\right) + 3x - 6 \\
-x^3 & & +4x & & \\ \hline
& -x^2 & +3x & -2 & \\
& x^2 & & -4 & \\ \hline
& & 3x & -6 &
\end{array}$$

$\Rightarrow \quad f(x) = x + \dfrac{x-3}{x^2-2x-3}$

$\Rightarrow$ AO : $\boxed{y = x}$

c) étude du signe de $\delta(x) = \dfrac{x-3}{x^2-2x-3}$

x	$-\infty$	-1	3	$+\infty$
$\delta(x)$	$-$	$+$	$+$	

f est dessus l'AO si $x \in]-1;3[\cup]3;+\infty[$
f est dessous l'AO si $x \in]-2;2[\cup]2;+\infty[$

$\Rightarrow \quad f(x) = x - 1 + \dfrac{3x-6}{x^2-4}$

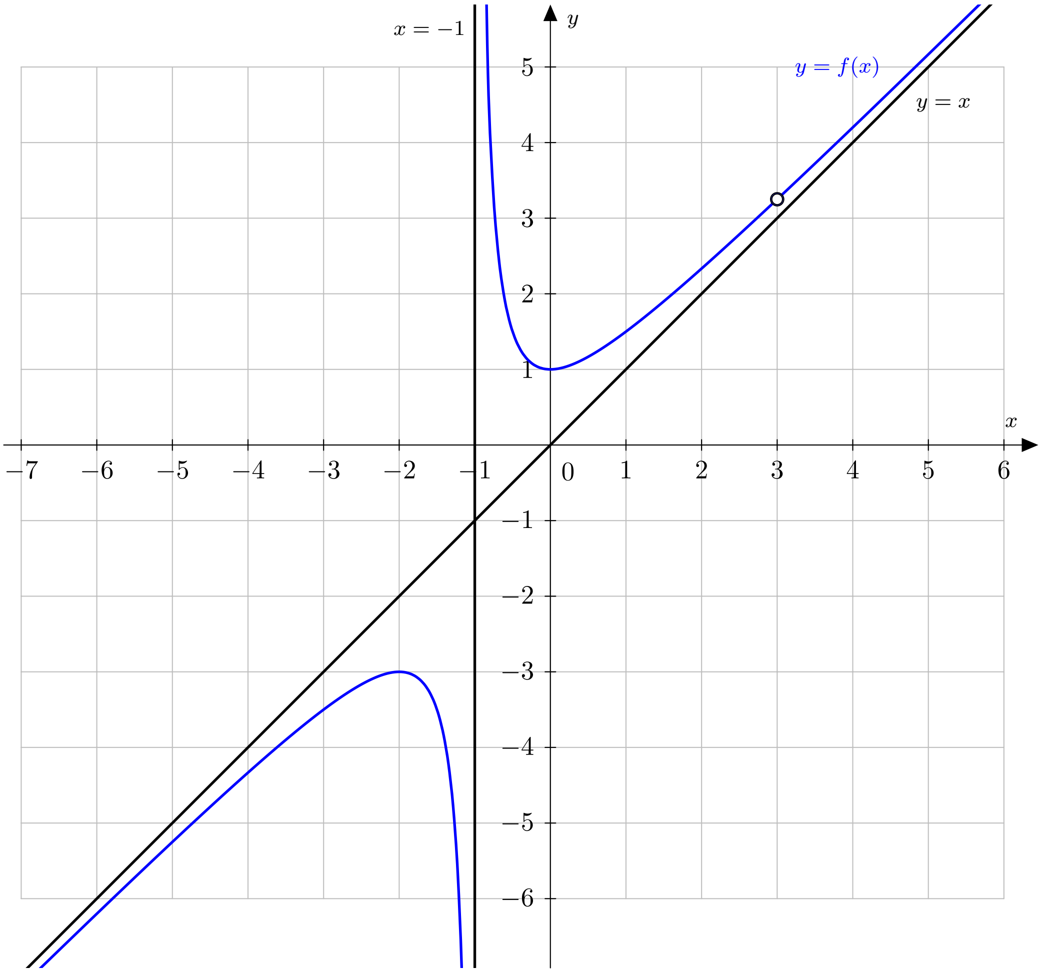
$\Rightarrow$ AO : $\boxed{y = x - 1}$

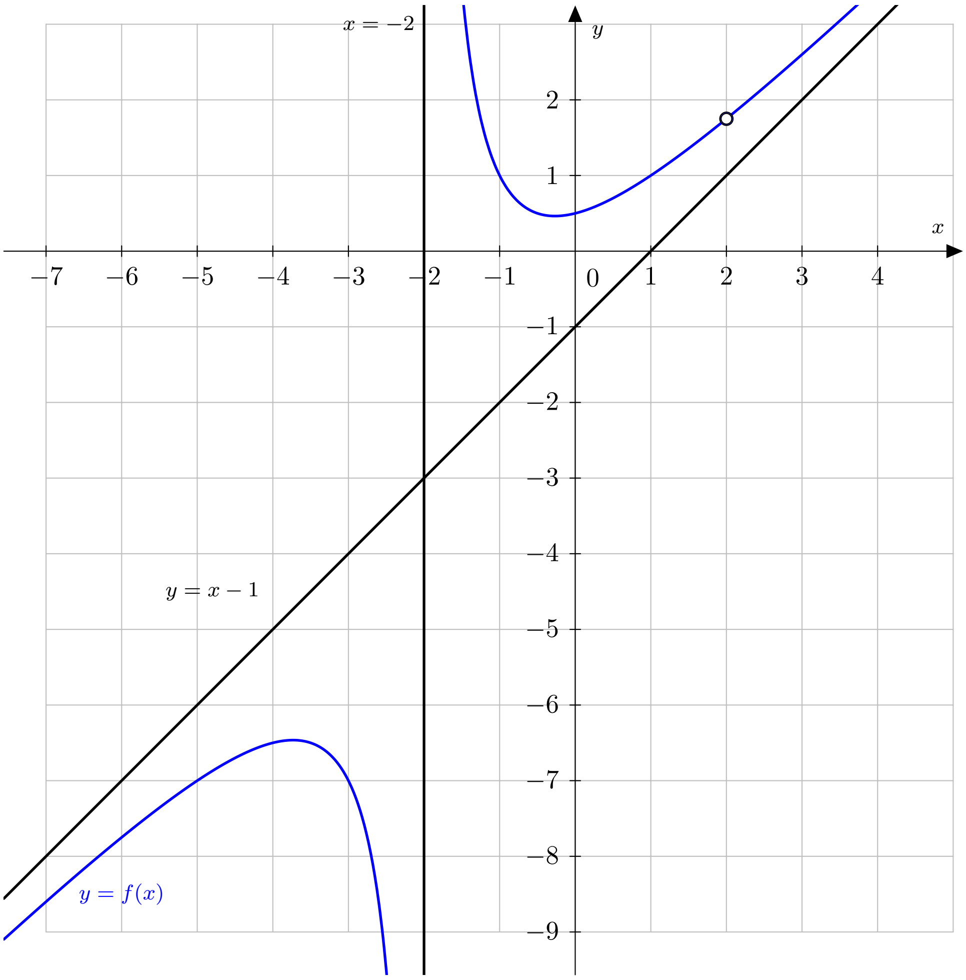
étude du signe de $\delta(x) = \dfrac{3x-6}{x^2-4}$

x	$-\infty$	-2	2	$+\infty$
$\delta(x)$	$-$	$+$	$+$	

f est dessus l'AO si $x \in]-\infty;-5[$
f est dessous l'AO si $x \in]-\infty,-2[$

d)





BONUS

$$\lim_{x\to-\infty}\left(2^{\frac{1}{x}}+2^x+\tfrac{x^2-1}{4x^2}\right)=1+0+\frac{1}{4}=\boxed{\frac{5}{4}}$$

$$\lim_{x\to-\infty}\left(3^{\frac{1}{x}}+3^x+\tfrac{x^3-1}{5x^3}\right)=1+0+\frac{1}{5}=\boxed{\frac{6}{5}}$$