Fonctions IV

Exercice 1.

a) $\lim\limits_{x\to 3} f(x) \overbrace{=}^{"\frac{0}{0}"\text{ (B-H)}} \lim\limits_{x\to 3} \dfrac{2}{3\,x^2-7} = \boxed{\dfrac{1}{10}}$

b) $\lim\limits_{x\to 0} f(x) \overbrace{=}^{"\frac{0}{0}"\text{ (B-H)}} \lim\limits_{x\to 0} \dfrac{\cos(x)}{4} = \boxed{\dfrac{1}{4}}$

$\lim\limits_{x\to 4} g(x) \overbrace{=}^{"\frac{0}{0}"\text{ (B-H)}} \lim\limits_{x\to 4} \dfrac{3\,x^2-15}{1} = \boxed{33}$

$\lim\limits_{x\to 0} i(x) \overbrace{=}^{"\frac{0}{0}"\text{ (B-H)}} \lim\limits_{x\to 0} \dfrac{\sin(x)}{5} = \boxed{0}$

Exercice 2.

a) $f(x) = 3x+5$
$g(x) = 2x^2-5x-5$

$f(-1) = 2 = g(-1) \quad \Rightarrow \boxed{I(-1;2)}$

b) $f'(x) = 3 \qquad g'(x) = 4x-5$

$m_1 = f'(-1) = 3 \qquad m_2 = g'(-1) = -9$

$\tan(\alpha) = \left|\dfrac{-9-3}{1-27}\right| = \dfrac{6}{13} \Rightarrow \boxed{\alpha \cong 24,78°}$

$f(x) = 4x-3$
$g(x) = 3x^2-2x-12$

$f(-1) = -7 = g(-1) \quad \Rightarrow \boxed{I(-1;-7)}$

$f'(x) = 4 \qquad g'(x) = 6x-2$

$,m_1 = f'(-1) = 4 \qquad m_2 = g'(-1) = -8$

$\tan(\alpha) = \left|\dfrac{-8-4}{1-32}\right| = \dfrac{12}{31} \Rightarrow \boxed{\alpha \cong 21,16°}$

Exercice 3.

$\boxed{ED(f) = \mathbb{R} - \{-1\}}$

$\Rightarrow$ zéro : $\boxed{x=3}$

x	$-\infty$		-1		3		$+\infty$
$f(x)$		$-$	‖	$+$	0	$+$	

$\lim\limits_{\substack{x\to -1 \\ <}} f(x) \overbrace{=}^{"\frac{16}{0_-}"} -\infty$ et $\lim\limits_{\substack{x\to -1 \\ >}} f(x) \overbrace{=}^{"\frac{16}{0_+}"} +\infty$

$\Rightarrow \boxed{\text{AV} : x=-1}$

AO $\Rightarrow$ division polynomiale

$\boxed{ED(f) = \mathbb{R} - \{3\}}$

$\Rightarrow$ zéro : $\boxed{x=-1}$

x	$-\infty$		-1		3		$+\infty$
$f(x)$		$-$	0	$-$	‖	$+$	

$\lim\limits_{\substack{x\to 3 \\ <}} f(x) \overbrace{=}^{"\frac{16}{0_-}"} -\infty$ et $\lim\limits_{\substack{x\to 3 \\ >}} f(x) \overbrace{=}^{"\frac{16}{0_+}"} +\infty$

$\Rightarrow \boxed{\text{AV} : x=3}$

AO $\Rightarrow$ division polynomiale

$$\begin{array}{rl} x^2 - 6x + 9 & = (x+1)\,(x-7)\, + 16 \\ -\,x^2 \;\; -\,x & \\ \hline -\,7x + 9 & \\ 7x + 7 & \\ \hline 16 & \end{array}$$

$\Rightarrow$ $\boxed{\text{AO} : y = x - 7}$

étude du signe de $\delta(x) = \dfrac{16}{x+1}$

x	$-\infty$	-1	$+\infty$
$\delta(x)$	$-$	$\|$	$+$

f est dessus l'AO si $x \in]-1; +\infty[$
f est dessous l'AO si $x \in]-\infty; -1[$

$$f'(x) = \frac{(2x-6)\cdot(x+1) - (x^2 - 6x + 9)\cdot 1}{(x+1)^2}$$
$$= \frac{x^2 + 2x - 15}{(x+1)^2} = \frac{(x+5)(x-3)}{(x+1)^2}$$

$ED(f') = ED(f)$

x	$-\infty$		-5		-1		3		$+\infty$
$f'(x)$		$+$	0	$-$	$\|$	$-$	0	$+$	
$f(x)$		$\nearrow$	-16	$\searrow$	$\|$	$\searrow$	0	$\nearrow$	

$$\begin{array}{rl} x^2 + 2x \;\; + 1 & = (x-3)\,(x+5)\, + 16 \\ -\,x^2 + 3x & \\ \hline 5x \;\; + 1 & \\ -\,5x + 15 & \\ \hline 16 & \end{array}$$

$\Rightarrow$ $\boxed{\text{AO} : y = x + 5}$

étude du signe de $\delta(x) = \dfrac{16}{x-3}$

x	$-\infty$	3	$+\infty$
$\delta(x)$	$-$	$\|$	$+$

f est dessus l'AO si $x \in]3; +\infty[$
f est dessous l'AO si $x \in]-\infty; 3[$

$$f'(x) = \frac{(2x+2)\cdot(x-3) - (x^2 + 2x + 1)}{(x-3)^2}$$
$$= \frac{x^2 - 6x - 7}{(x-3)^2} = \frac{(x+1)(x-7)}{(x-3)^2}$$

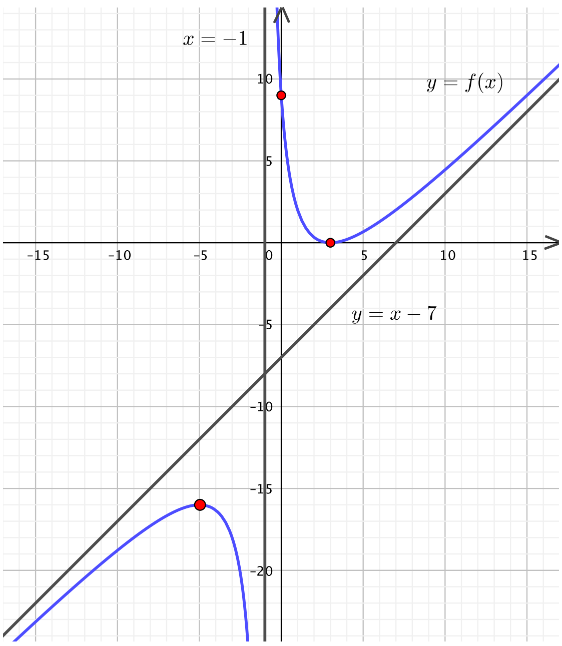
$ED(f') = ED(f)$

x	$-\infty$		-1		3		7		$+\infty$
$f'(x)$		$+$	0	$-$	$\|$	$-$	0	$+$	
$f(x)$		$\nearrow$	0	$\searrow$	$\|$	$\searrow$	16	$\nearrow$	

max : $(-5;-16)$

min : $(3;0)$

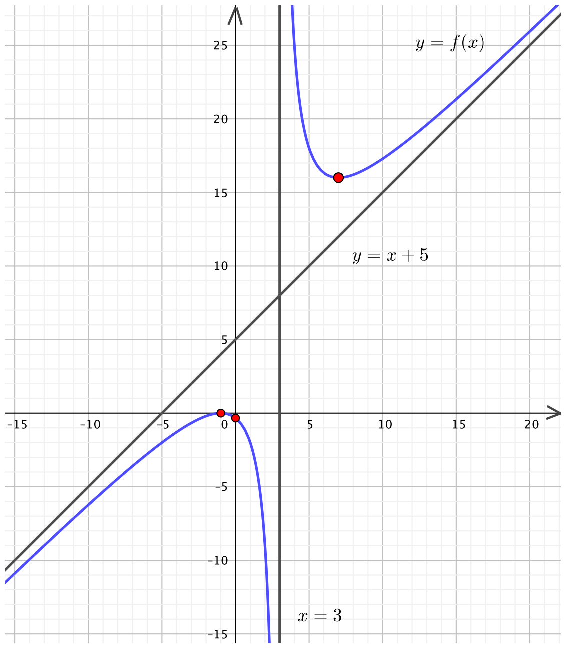
pt. part. : $(0;9)$



max : $(-1;0)$

min : $(7;16)$

pt. part. : $(0;-\frac{1}{3})$



Exercice 4.

a) y : largeur du paquet en cm

$$f(x;y) = 2x+2y+4\cdot 25+30 = 2x+2y+130$$

$$V = 25xy = 10000 \quad \Rightarrow \quad y = \frac{400}{x}$$

$$f(x) = 2x + \frac{800}{x} + 130 = \frac{2x^2+130x+800}{x}$$

$$ED(f) =]0;+\infty[$$

b) $$f'(x) = 2 - \frac{800}{x^2} = \frac{2x^2-800}{x^2}$$

$$\Rightarrow f'(x) = \frac{2(x-20)(x+20)}{x^2}$$

x	$-\infty$	0		20		$+\infty$
$f'(x)$		‖	$-$	0	$+$	
$f(x)$		‖	$\searrow$	210	$\nearrow$	

$\Rightarrow$ dimensions : 25 cm, 20 cm et 20 cm

$\Rightarrow$ longueur minimale : 210 cm

y : largeur du paquet en cm

$$f(x;y) = 2x+2y+4\cdot 40+30 = 2x+2y+190$$

$$V = 40xy = 9000 \quad \Rightarrow \quad y = \frac{225}{x}$$

$$f(x) = 2x + \frac{450}{x} + 190 = \frac{2x^2+190x+450}{x}$$

$$ED(f) =]0;+\infty[$$

$$f'(x) = 2 - \frac{450}{x^2} = \frac{2x^2-450}{x^2}$$

$$\Rightarrow f'(x) = \frac{2(x-15)(x+15)}{x^2}$$

x	$-\infty$	0		15		$+\infty$
$f'(x)$		‖	$-$	0	$+$	
$f(x)$		‖	$\searrow$	250	$\nearrow$	

$\Rightarrow$ dimensions : 40 cm, 15 cm et 15 cm

$\Rightarrow$ longueur minimale : 250 cm

BONUS

$$f(x) = \frac{5}{4}x^4 - \frac{2}{3}x^3 + \frac{7}{2}x^2 - 8x + c$$

$$f(0) = 3 \quad \Rightarrow \quad c = 3$$

$$\boxed{f(x) = \frac{5}{4}x^4 - \frac{2}{3}x^3 + \frac{7}{2}x^2 - 8x + 3}$$

$$f(x) = \frac{3}{4}x^4 - \frac{5}{3}x^3 + \frac{9}{2}x^2 + 7x + c$$

$$f(0) = 8 \quad \Rightarrow \quad c = 8$$

$$\boxed{f(x) = \frac{3}{4}x^4 - \frac{5}{3}x^3 + \frac{9}{2}x^2 + 7x + 8}$$