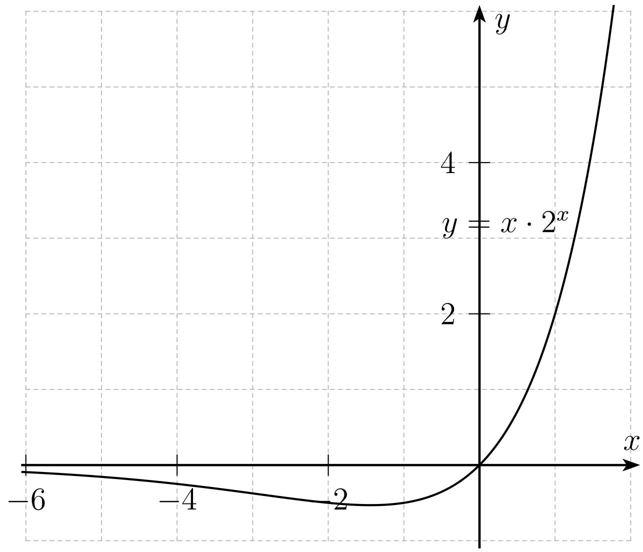
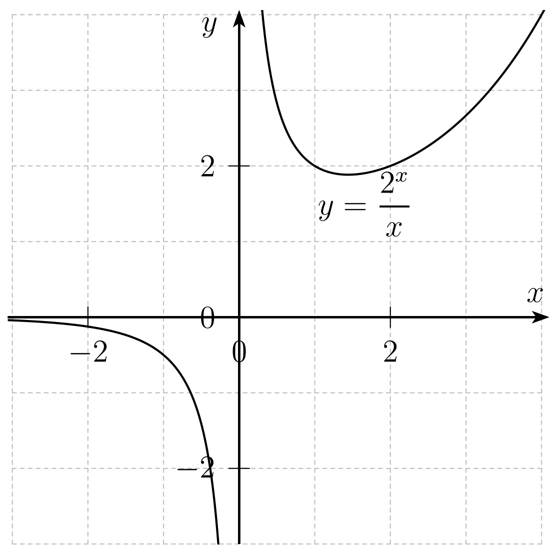
1.2.9

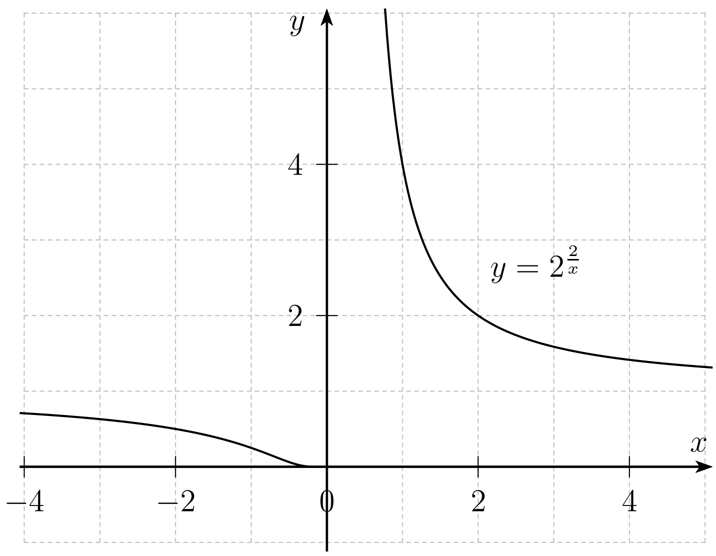
a)



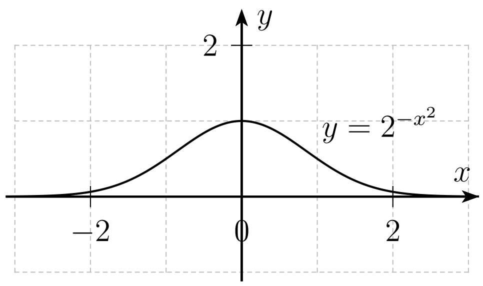
b)



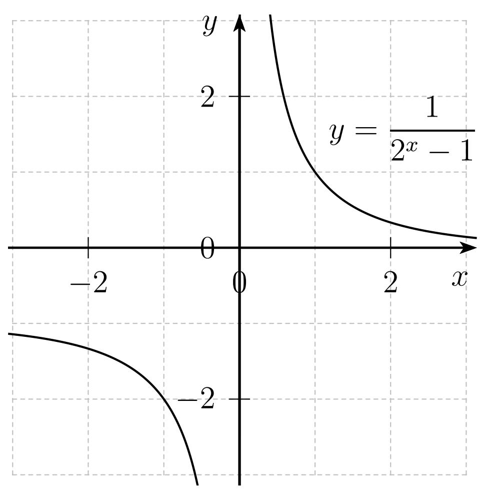
c)



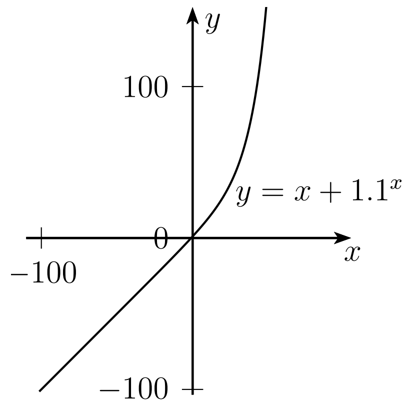
d)



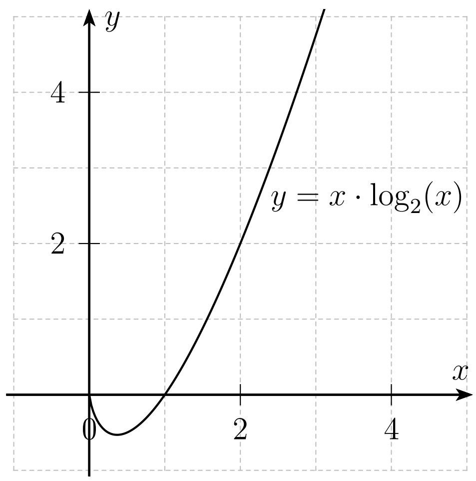
e)



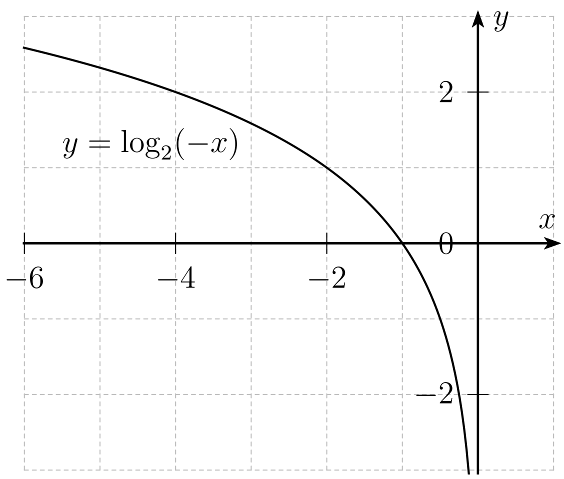
f)



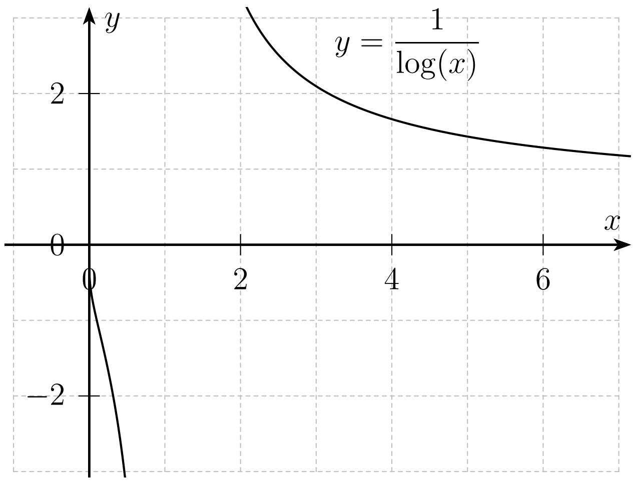
g)



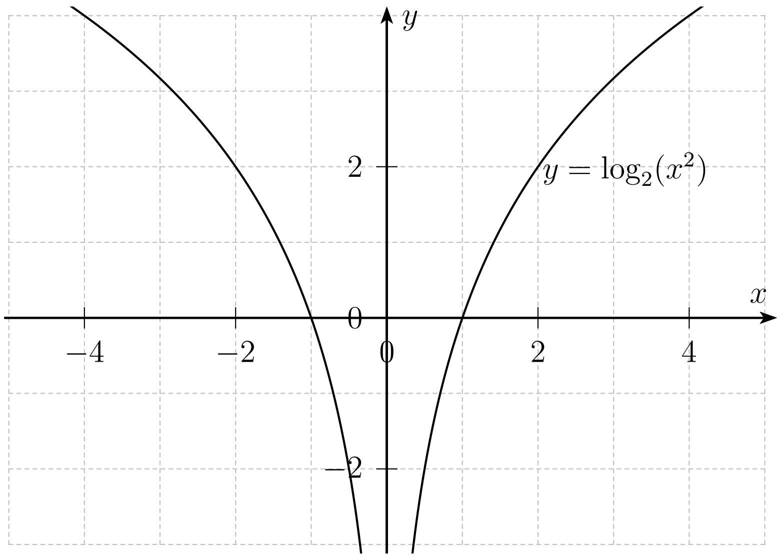
h)



i)



j)



1.2.10

a) $10^x - 9 = 0 \quad \Rightarrow \quad 10^x = 9 \quad \Rightarrow \quad x = \log(9) \quad \Rightarrow \quad ED(f) = \mathbb{R} - \{\log(9)\}$

b) $\dfrac{x^2-1}{x+3} > 0 \quad \Rightarrow \quad \dfrac{(x-1)(x+1)}{x+3} > 0 \quad \Rightarrow \quad ED(f) =]-3; -1[\cup]1; +\infty[$

c) $x^3 + 2x^2 - 3 > 0 \quad \Rightarrow \quad \begin{array}{c|cccc} & 1 & 2 & 0 & -3 \\ 1 & & 1 & 3 & 3 \\ \hline & 1 & 3 & 3 & 0 \end{array} \quad \Rightarrow \quad (x-1)\underbrace{(x^2+3x+3)}_{\Delta<0} > 0$

$\Rightarrow \quad ED(f) =]1; +\infty[$

d) $x^2 - 1 > 0 \quad \Rightarrow \quad (x-1)(x+1) > 0$

$x^2 - 1 = 1 \quad \Rightarrow \quad x^2 = 2 \quad \Rightarrow \quad x = \pm\sqrt{2}$

$\Rightarrow \quad ED(f) =]-\infty; -\sqrt{2}[\cup]-\sqrt{2}; -1[\cup]1; \sqrt{2}[\cup]\sqrt{2}; +\infty[$

1.2.11

a) $-x^2 + 4x + 22 > 0 \quad \Rightarrow \quad x^2 - 4x - 22 < 0 \qquad \Delta = 104$

$x = \dfrac{4 \pm 2\sqrt{26}}{2} = 2 \pm \sqrt{26} \quad \Rightarrow \quad ED(f) =]2 - \sqrt{26}; 2 + \sqrt{26}[$

zéros : $-x^2 + 4x + 22 = 1 \quad \Rightarrow \quad x^2 - 4x - 21 = 0 \quad \Rightarrow \quad (x-7)(x+3) = 0$

$\Rightarrow \quad x_1 = -3$ et $x_2 = 7$

x	$-\infty$	$2-\sqrt{26}$		-3		7		$2+\sqrt{26}$	$+\infty$
$f(x)$	(hachuré)	|	$-$	0	$+$	0	$-$	|	(hachuré)

b) $ED(f) = \mathbb{R}$

zéro : $12 - 10^{3-x} = 0 \quad \Rightarrow \quad 10^{3-x} = 12 \quad \Rightarrow \quad 3 - x = \log(12)$

$\Rightarrow \quad x = 3 - \log(12)$

x	$-\infty$		$3-\log(12)$		$+\infty$
$f(x)$		$-$	0	$+$	

c) $\dfrac{2x}{x-1} > 0 \quad \Rightarrow \quad ED(f) =]-\infty; 0[\cup]1; +\infty[$

zéro : $\dfrac{2x}{x-1} = 1 \quad \Rightarrow \quad 2x = x - 1 \quad \Rightarrow \quad x = -1$

x	$-\infty$		-1		0		1		$+\infty$
$f(x)$		$+$	0	$-$	|	(hachuré)	|	$+$	

1.2.12

a) $s(5) = \dfrac{20\ln(6)+5}{6} \simeq 6.81$

b) $ED(s) = [1; +\infty[$

$$s'(t) = \frac{\left(\frac{20}{t+1}+1\right)\cdot(t+1) - (20\ln(t+1)+t)\cdot 1}{(t+1)^2} = \frac{21 - 20\ln(t+1)}{(t+1)^2}$$

zéro de s' : $\ln(t+1) = \dfrac{21}{20} \quad \Rightarrow \quad t+1 = e^{\frac{21}{20}} \quad \Rightarrow \quad t = e^{\frac{21}{20}} - 1 \simeq 1.86$

t	1	1.86	$+\infty$
$s'(t)$	+	0	−
$s(t)$	↗	Max	↘

$\Rightarrow$ pendant le mois de février l'indice de satisfaction est maximal

c) $$\lim_{t\to+\infty} s(t) = \lim_{t\to+\infty} \frac{20\ln(t+1)+t}{t+1} \underbrace{=}_{\text{L'Hospital}} \lim_{t\to+\infty} \frac{\overbrace{\frac{20}{t+1}}^{\to 0}+1}{1} = 1$$

1.2.13

a) $d(0) = \dfrac{0+2}{2} = 1$

b) $d(20) = \dfrac{20\cdot e^{-\frac{2}{3}}+2}{2} = 10\cdot e^{-\frac{2}{3}} + 1 \simeq 6.13$

c) $ED(d) = [0; 45]$ $\qquad d'(t) = \dfrac{1}{2}e^{-\frac{t}{30}} + \dfrac{1}{2}t\cdot\left(-\dfrac{1}{30}\right)\cdot e^{-\frac{t}{30}} = -\dfrac{1}{2}e^{-\frac{t}{30}}\left(\dfrac{t}{30}-1\right)$

t	0	30	45
$d'(t)$	+	0	−
$d(t)$	↗	Max	↘

$d(30) = \dfrac{30\cdot e^{-1}+2}{2} = \dfrac{15}{e} + 1 \simeq 6.52$

$\Rightarrow$ après 30 minutes le degré est maximal et vaut environ 6.52