Limites de fonctions

2.4.1 a) b, b, b ; b) b, b, b ; c) b, $f(a)$, - ; d) b, c, -.

2.4.2

a) $\lim\limits_{x\to 1}(4x^3-2x^2+x-1)=4-2+1-1=2$

b) $\lim\limits_{x\to -2}(x^2-5x)=4+10=14$

c) $\lim\limits_{x\to 0}\dfrac{x+3x^2}{x+1}=\dfrac{0}{1}=0$

d) $\lim\limits_{x\to 4}(-5)=-5$

e) $\lim\limits_{x\to 3}\sqrt{x^2-5}=\sqrt{9-5}=2$

f) $\lim\limits_{x\to 1}\dfrac{x^2+2x+1}{x^3+x^2+x}=\dfrac{1+2+1}{1+1+1}=\dfrac{4}{3}$

g) $\lim\limits_{x\to \frac{\pi}{2}}\cos\left(x+\dfrac{\pi}{2}\right)=\cos(\pi)=-1$

h) $\lim\limits_{x\to 0}\dfrac{\sin(x)+1}{2-\tan(x)}=\dfrac{1}{2}$

2.4.3

a) $\lim\limits_{x\to 3}\dfrac{x-3}{2x-6}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 3}\dfrac{x-3}{2(x-3)}=\lim\limits_{x\to 3}\dfrac{1}{2}=\dfrac{1}{2}$

b) $\lim\limits_{x\to 0}\dfrac{100x^2}{x}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 0}\dfrac{100x}{1}=0$

c) $\lim\limits_{x\to 2}\dfrac{(x-2)(x+1)}{(x+4)(x-2)}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 2}\dfrac{x+1}{x+4}=\dfrac{3}{6}=\dfrac{1}{2}$

d) $\lim\limits_{x\to 1}\dfrac{x^2-1}{x-1}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 1}\dfrac{(x-1)(x+1)}{x-1}=\lim\limits_{x\to 1}\dfrac{x+1}{1}=2$

e) $\lim\limits_{x\to 5}\dfrac{x^2+2x-15}{x^2+8x+15}=\dfrac{20}{80}=\dfrac{1}{4}$

f) $\lim\limits_{x\to 4}\dfrac{x^2-4x}{x^2-16}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 4}\dfrac{x(x-4)}{(x-4)(x+4)}=\lim\limits_{x\to 4}\dfrac{x}{x+4}=\dfrac{4}{8}=\dfrac{1}{2}$

g) $\lim\limits_{x\to 1}\dfrac{x^2-x}{x^3-4x^2-7x+10}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 1}\dfrac{x(x-1)}{x(x^2-3x-10)}=\lim\limits_{x\to 1}\dfrac{x}{x^2-3x-10}=-\dfrac{1}{12}$

$$\begin{array}{r|rrr|r}
 & 1 & -4 & -7 & 10 \\
1 & & 1 & -3 & -10 \\
\hline
 & 1 & -3 & -10 & 0
\end{array}$$

h) $\lim\limits_{x\to 1}\dfrac{2x^3-3x^2+1}{x^2+3x-4}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 1}\dfrac{(x-1)(2x^2-x-1)}{(x+4)(x-1)}=\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{x+4}=\dfrac{0}{5}=0$

$$\begin{array}{c|cccc}
 & 2 & -3 & 0 & 1 \\
1 & & 2 & -1 & -1 \\
\hline
 & 2 & -1 & -1 & \multicolumn{1}{|c}{0}
\end{array}$$

2.4.4

a) $\lim\limits_{x\to 16}\dfrac{x-16}{\sqrt{x}-4}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 16}\dfrac{x-16}{\sqrt{x}-4}\cdot\dfrac{\sqrt{x}+4}{\sqrt{x}+4}=\lim\limits_{x\to 16}\dfrac{(x-16)(\sqrt{x}+4)}{x-16}=\lim\limits_{x\to 16}(\sqrt{x}+4)=8$

b) $\lim\limits_{x\to 25}\dfrac{\sqrt{x}-5}{x-25}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 25}\dfrac{\sqrt{x}-5}{(\sqrt{x}-5)(\sqrt{x}+5)}=\lim\limits_{x\to 25}\dfrac{1}{\sqrt{x}+5}=\dfrac{1}{10}$

c) $\lim\limits_{h\to 0}\dfrac{4-\sqrt{16+h}}{h}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{h\to 0}\dfrac{(4-\sqrt{16+h})(4+\sqrt{16+h})}{h(4+\sqrt{16+h})}=\lim\limits_{h\to 0}\dfrac{-h}{h(4+\sqrt{16+h})}$

$=\lim\limits_{h\to 0}\dfrac{-1}{4+\sqrt{16+h}}=-\dfrac{1}{8}$

d) $\lim\limits_{x\to 0}\dfrac{x^2}{\sqrt{x^2+1}-1}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 0}\dfrac{x^2(\sqrt{x^2+1}+1)}{(\sqrt{x^2+1}-1)(\sqrt{x^2+1}+1)}=\lim\limits_{x\to 0}\dfrac{x^2(\sqrt{x^2+1}+1)}{x^2}$

$=\lim\limits_{x\to 0}(\sqrt{x^2+1}+1)=2$

e) $\lim\limits_{x\to 5}\dfrac{x-5}{\sqrt{2x-1}-3}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 5}\dfrac{(x-5)(\sqrt{2x-1}+3)}{(\sqrt{2x-1}-3)(\sqrt{2x-1}+3)}=\lim\limits_{x\to 5}\dfrac{(x-5)(\sqrt{2x-1}+3)}{2x-10}$

$=\lim\limits_{x\to 5}\dfrac{(x-5)(\sqrt{2x-1}+3)}{2(x-5)}=\lim\limits_{x\to 5}\dfrac{\sqrt{2x-1}+3}{2}=\dfrac{6}{2}=3$

f) $\lim\limits_{t\to 2}\dfrac{1+\sqrt{t-2}}{t}=\dfrac{1}{2}$

g) $\lim\limits_{x\to 2}\dfrac{x-2}{\sqrt{x+1}-\sqrt{2x-1}}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to 2}\dfrac{(x-2)(\sqrt{x+1}+\sqrt{2x-1})}{(\sqrt{x+1}-\sqrt{2x-1})(\sqrt{x+1}+\sqrt{2x-1})}$

$=\lim\limits_{x\to 2}\dfrac{(x-2)(\sqrt{x+1}+\sqrt{2x-1})}{-(x-2)}=\lim\limits_{x\to 2}\dfrac{\sqrt{x+1}+\sqrt{2x-1}}{-1}=\dfrac{2\sqrt{3}}{-1}=-2\sqrt{3}$

h) $\lim\limits_{x\to -2}\dfrac{x+\sqrt{x+6}}{x+\sqrt{2-x}}\overbrace{=}^{\frac{0}{0}\text{ fi}}\lim\limits_{x\to -2}\dfrac{(x+\sqrt{x+6})(x-\sqrt{2-x})}{(x+\sqrt{2-x})(x-\sqrt{2-x})}=\lim\limits_{x\to -2}\dfrac{(x+\sqrt{x+6})(x-\sqrt{2-x})}{x^2+x-2}$

$=\lim\limits_{x\to -2}\dfrac{(x+\sqrt{x+6})(x-\sqrt{x+6})(x-\sqrt{2-x})}{(x+2)(x-1)(x-\sqrt{x+6})}=\lim\limits_{x\to -2}\dfrac{(x^2-x-6)(x-\sqrt{2-x})}{(x+2)(x-1)(x-\sqrt{x+6})}$

$=\lim\limits_{x\to -2}\dfrac{(x-3)(x+2)(x-\sqrt{2-x})}{(x+2)(x-1)(x-\sqrt{x+6})}=\lim\limits_{x\to -2}\dfrac{(x-3)(x-\sqrt{2-x})}{(x-1)(x-\sqrt{x+6})}=\dfrac{20}{12}=\dfrac{5}{3}$