

Exponentielles et logarithmes

Exercice 1.

$$a) 13^{3x+6} = 13^2 \Leftrightarrow 3x + 6 = 2$$

$$\Leftrightarrow 3x = -4 \Rightarrow S = \left\{ -\frac{4}{3} \right\}$$

$$17^{5x+1} = 17^2 \Leftrightarrow 5x + 1 = 2$$

$$\Leftrightarrow 5x = 1 \Rightarrow S = \left\{ \frac{1}{5} \right\}$$

$$b) 5^{2x} - 4 \cdot 5^x - 5 = 0 \Rightarrow y^2 - 4y - 5 = 0$$

$$(y-5)(y+1) = 0 \Rightarrow y = 5 \text{ ou } y = -1$$

$$\Rightarrow 5^x = 5 \text{ ou } 5^x = -1 \text{ (pas de sol.)}$$

$$\Rightarrow x = 1 \Rightarrow S = \{1\}$$

$$3^{2x} - 4 \cdot 3^x + 3 = 0 \Rightarrow y^2 - 4y + 3 = 0$$

$$(y-3)(y-1) = 0 \Rightarrow y = 3 \text{ ou } y = 1$$

$$\Rightarrow 3^x = 3 \text{ ou } 3^x = 1$$

$$\Rightarrow x = 1 \text{ ou } x = 0 \Rightarrow S = \{0; 1\}$$

$$c) \log(x) = \log(\sqrt{81}) - \log(12)$$

$$ED(f) = \mathbb{R}_+^*$$

$$\Rightarrow \log(x) = \log\left(\frac{9}{12}\right)$$

$$\Rightarrow x = \frac{9}{12} = \frac{3}{4} \Rightarrow S = \left\{ \frac{3}{4} \right\}$$

$$\log(x) = \log(\sqrt{49}) - \log(21)$$

$$ED(f) = \mathbb{R}_+^*$$

$$\Rightarrow \log(x) = \log\left(\frac{7}{21}\right)$$

$$\Rightarrow x = \frac{7}{21} = \frac{1}{3} \Rightarrow S = \left\{ \frac{1}{3} \right\}$$

Exercice 2.

$$a) x^2 - 3 > 0$$

x	$-\infty$	$-\sqrt{3}$	$\sqrt{3}$	$+\infty$	
$x^2 - 3$	+	0	-	0	+

$$\Rightarrow ED(f) =] - \infty; -\sqrt{3}[\cup] \sqrt{3}; +\infty[$$

$$x^2 - 15 > 0$$

x	$-\infty$	$-\sqrt{15}$	$\sqrt{15}$	$+\infty$	
$x^2 - 15$	+	0	-	0	+

$$\Rightarrow ED(f) =] - \infty; -\sqrt{15}[\cup] \sqrt{15}; +\infty[$$

$$\text{b) zéro : } x^2 - 3 = 1 \Leftrightarrow x^2 - 4 = 0$$

$$\Rightarrow x = \pm 2$$

x	$-\infty$	-2	$-\sqrt{3}$	$\sqrt{3}$	2	$+\infty$	
$f(x)$	+	0	-		-	0	+

$$\text{zéro : } x^2 - 15 = 1 \Leftrightarrow x^2 - 16 = 0$$

$$\Rightarrow x = \pm 4$$

x	$-\infty$	-4	$-\sqrt{15}$	$\sqrt{15}$	4	$+\infty$	
$f(x)$	+	0	-		-	0	+

Exercice 3.

$$\text{a) } h(13) \simeq \boxed{2.157 \text{ m}}$$

$$h(9) \simeq \boxed{1.003 \text{ m}}$$

$$\text{b) } 18.5 = \frac{35}{1 + 205 \cdot e^{-0.2t}}$$

$$\Rightarrow e^{-0.2t} = \left(\frac{35}{18.5} - 1 \right) \cdot \frac{1}{205} = \frac{33}{7585}$$

$$\Rightarrow t = -\frac{1}{0.2} \cdot \ln \left(\frac{33}{7585} \right) \simeq 27.19$$

$$\Rightarrow \boxed{27 \text{ ans et 2 mois}}$$

$$16.5 = \frac{35}{1 + 205 \cdot e^{-0.2t}}$$

$$\Rightarrow e^{-0.2t} = \left(\frac{35}{16.5} - 1 \right) \cdot \frac{1}{205} = \frac{37}{6765}$$

$$\Rightarrow t = -\frac{1}{0.2} \cdot \ln \left(\frac{37}{6765} \right) \simeq 26.04$$

$$\Rightarrow \boxed{26 \text{ ans}}$$

Exercice 4.

$$\text{a) } N(t) = \boxed{100 \cdot 8^{\frac{t}{3}}}$$

$$N(t) = \boxed{200 \cdot 5^{\frac{t}{2}}}$$

$$\text{b) } N(6) = 100 \cdot 8^{\frac{6}{3}} = \boxed{6'400 \text{ bactéries}}$$

$$N(7) = 200 \cdot 5^{\frac{7}{2}} \simeq \boxed{55'902 \text{ bactéries}}$$

$$\text{c) } 1000 = 100 \cdot 8^{\frac{t}{3}} \Leftrightarrow 10 = 8^{\frac{t}{3}}$$

$$3'000 = 200 \cdot 5^{\frac{t}{2}} \Leftrightarrow 15 = 5^{\frac{t}{2}}$$

$$\Leftrightarrow t = 3 \cdot \log_8(10) \simeq 3,32 \Rightarrow \boxed{3 \text{ h et 19 min}}$$

$$\Leftrightarrow t = 5 \cdot \log_5(15) \simeq 3,37 \Rightarrow \boxed{3 \text{ h et 22 min}}$$

Exercice 5.

$$\text{a) } C(n) = 1'000 \cdot 1,05^n$$

$$\text{b) } 2'000 = 1'000 \cdot 1,05^n \Leftrightarrow 2 = 1,05^n$$

$$\Leftrightarrow n = \log_{1,05}(2) \simeq 14,21 \quad (15 \text{ ans})$$

$$\Rightarrow \text{en l'an } 2035$$

$$\text{c) } 3'000 = 1'000 \cdot \left(1 + \frac{t}{100}\right)^{15}$$

$$\Leftrightarrow \left(1 + \frac{t}{100}\right)^{15} = 3$$

$$\Leftrightarrow 1 + \frac{t}{100} = \sqrt[15]{3} \Leftrightarrow \frac{t}{100} = \sqrt[15]{3} - 1$$

$$\Rightarrow t \simeq 7,6 \Rightarrow 7,6 \%$$

$$C(n) = 1'000 \cdot 1,04^n$$

$$4'500 = 1'500 \cdot 1,04^n \Leftrightarrow 3 = 1,04^n$$

$$\Leftrightarrow n = \log_{1,04}(3) \simeq 28,01 \quad (29 \text{ ans})$$

$$\Rightarrow \text{en l'an } 2044$$

$$6'000 = 1'500 \cdot \left(1 + \frac{t}{100}\right)^{20}$$

$$\Leftrightarrow \left(1 + \frac{t}{100}\right)^{20} = 4$$

$$\Leftrightarrow 1 + \frac{t}{100} = \sqrt[20]{4} \Leftrightarrow \frac{t}{100} = \sqrt[20]{4} - 1$$

$$\Rightarrow t \simeq 7,18 \Rightarrow 7,18 \%$$