Applications de l'intégrale

Exercice 1.

pts d'int. : $2x^2 - x + 3 = x^2 - 4x + 7$

$x^2 + 3x - 4 = 0 \quad \Rightarrow \quad (x+4)(x-1) = 0$

$\Rightarrow \quad I_1(-4; 39)$ et $I_2(1; 4)$

$$\int_{-4}^{1} (x^2 + 3x - 4)\,\mathrm{d}x = \frac{1}{3}\,x^3 + \frac{3}{2}\,x^2 - 4\,x \Big|_{-4}^{1}$$

$$= -\frac{13}{6} - \left(\frac{56}{3}\right) = -\frac{125}{6}$$

$$\Rightarrow \quad \mathcal{A} = \boxed{\frac{125}{6}\ \mathrm{u}^2}$$

pts d'int. : $2x^2 + 4x - 5 = x^2 - x + 1$

$x^2 + 5x - 6 = 0 \quad \Rightarrow \quad (x+6)(x-1) = 0$

$\Rightarrow \quad I_1(-6; 43)$ et $I_2(1; 1)$

$$\int_{-6}^{1} (x^2 + 5x - 6)\,\mathrm{d}x = \frac{1}{3}\,x^3 + \frac{5}{2}\,x^2 - 6\,x \Big|_{-6}^{1}$$

$$= -\frac{19}{6} - (54) = -\frac{343}{6}$$

$$\Rightarrow \quad \mathcal{A} = \boxed{\frac{343}{6}\ \mathrm{u}^2}$$

Exercice 2.

zéros : $x^3 - 5x^2 + 4x = 0$

$x(x^2 - 5x + 4) = 0 \quad \Rightarrow \quad x(x-4)(x-1) = 0$

$\Rightarrow \quad x_1 = 0,\ x_2 = 1,\ x_3 = 4$

$$\int_{0}^{1} (x^3 - 5x^2 + 4x)\,\mathrm{d}x = \frac{1}{4}\,x^4 - \frac{5}{3}\,x^3 + 2\,x^2 \Big|_{0}^{1}$$

$$= \frac{7}{12} - 0 = \frac{7}{12}$$

$$\int_{1}^{4} (x^3 - 5x^2 + 4x)\,\mathrm{d}x = \frac{1}{4}\,x^4 - \frac{5}{3}\,x^3 + 2\,x^2 \Big|_{1}^{4}$$

$$= -\frac{32}{3} - \frac{7}{12} = -\frac{45}{4}$$

$$\Rightarrow \quad \mathcal{A} = \frac{7}{12} + \frac{45}{4} = \boxed{\frac{71}{6}\ \mathrm{u}^2}$$

zéros : $x^3 - 4x^2 + 3x = 0$

$x(x^2 - 4x + 3) = 0 \quad \Rightarrow \quad x(x-3)(x-1) = 0$

$\Rightarrow \quad x_1 = 0,\ x_2 = 1,\ x_3 = 3$

$$\int_{0}^{1} (x^3 - 4x^2 + 3x)\,\mathrm{d}x = \frac{1}{4}\,x^4 - \frac{4}{3}\,x^3 + \frac{3}{2}\,x^2 \Big|_{0}^{1}$$

$$= \frac{5}{12} - 0 = \frac{5}{12}$$

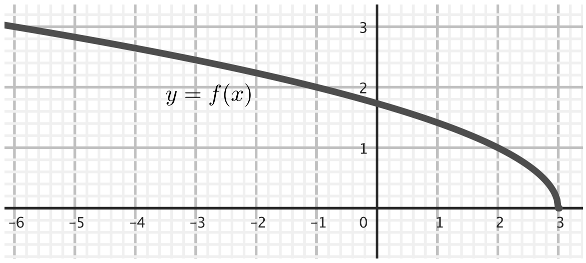
$$\int_{1}^{3} (x^3 - 4x^2 + 3x)\,\mathrm{d}x = \frac{1}{4}\,x^4 - \frac{4}{3}\,x^3 + \frac{3}{2}\,x^2 \Big|_{1}^{3}$$

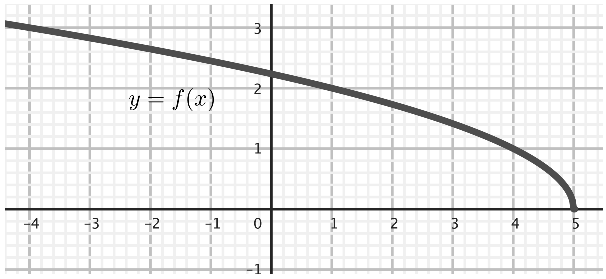
$$= -\frac{9}{4} - \frac{5}{12} = -\frac{8}{3}$$

$$\Rightarrow \quad \mathcal{A} = \frac{5}{12} + \frac{8}{3} = \boxed{\frac{37}{12}\ \mathrm{u}^2}$$

Exercice 3.

a)





b) $f(3) = 0 \quad \Rightarrow \quad x = 3$ est le zéro de f

$$\mathcal{A} = \int_{-6}^{3} \sqrt{3-x}\ \mathrm{d}x = -\int_{-6}^{3} (-1)(3-x)^{\frac{1}{2}}\ \mathrm{d}x$$

$$= -\frac{2}{3}(-x+3)^{\frac{3}{2}}\Big|_{-6}^{3} = \boxed{18\ \mathrm{u}^2}$$

$f(5) = 0 \quad \Rightarrow \quad x = 5$ est le zéro de f

$$A = \int_{-4}^{5} \sqrt{5-x}\ \mathrm{d}x = -\int_{-4}^{5} (-1)(5-x)^{\frac{1}{2}}\ \mathrm{d}x$$

$$= -\frac{2}{3}(-x+5)^{\frac{3}{2}}\Big|_{-4}^{5} = \boxed{18\ \mathrm{u}^2}$$

c) $$V = \pi \int_{-6}^{3} (3-x)\ \mathrm{d}x = \pi \left[-\frac{1}{2}x^2 + 3x\right]_{-6}^{3}$$

$$= \boxed{\frac{81}{2}\pi\ \mathrm{u}^3}$$

$$V = \pi \int_{-4}^{5} (5-x)\ \mathrm{d}x = \pi \left[-\frac{1}{2}x^2 + 5x\right]_{-4}^{5}$$

$$= \boxed{\frac{81}{2}\pi\ \mathrm{u}^3}$$

Exercice 4.

a) $f(-1) = g(-1) = 1$

$\Rightarrow$ $\boxed{A(-1;1) \text{ est un point d'int.}}$

$f(2) = g(2) = 1$

$\Rightarrow$ $\boxed{A(2;1) \text{ est un point d'int.}}$

b) $\displaystyle\int_{-1}^{0} \frac{4}{(x-1)^2}\,\mathrm{d}x = 4\int_{-1}^{0}(x-1)^{-2}\,\mathrm{d}x =$

$\displaystyle 4\left(-\frac{1}{x-1}\right)\Bigg|_{-1}^{0} = 2$

$\displaystyle\int_{-1}^{0}(-x)\,\mathrm{d}x = -\frac{1}{2}\,x^2\Big|_{-1}^{0} = \frac{1}{2}$

$\displaystyle\Rightarrow \quad \text{aire de } D = 2 - \frac{1}{2} = \frac{3}{2}$

$\displaystyle\int_{-3}^{0} \frac{4}{(x-1)^2}\,\mathrm{d}x = 4\left(-\frac{1}{x-1}\right)\Bigg|_{-3}^{0} = 3$

$\displaystyle\Rightarrow \quad \boxed{\text{aire de } E = 3 - \frac{3}{2} = \frac{3}{2} = \text{aire de } D}$

$\displaystyle\int_{0}^{2} \frac{16}{(x+2)^2}\,\mathrm{d}x = 16\int_{0}^{2}(x+2)^{-2}\,\mathrm{d}x =$

$\displaystyle 16\left(-\frac{1}{x+2}\right)\Bigg|_{0}^{2} = 4$

$\displaystyle\int_{0}^{2}\frac{1}{2}x\,\mathrm{d}x = \frac{1}{4}\,x^2\Big|_{0}^{2} = 1$

$\Rightarrow \quad \text{aire de } D = 4 - 1 = 3$

$\displaystyle\int_{0}^{6} \frac{16}{(x+2)^2}\,\mathrm{d}x = 16\left(-\frac{1}{x+2}\right)\Bigg|_{0}^{6} = 6$

$\Rightarrow \quad \boxed{\text{aire de } E = 6 - 3 = 3 = \text{aire de } D}$

c) $\displaystyle\pi\int_{-3}^{-1} \frac{16}{(x-1)^4}\,\mathrm{d}x = 16\pi\int_{-3}^{-1}(x-1)^{-4}\,\mathrm{d}x$

$\displaystyle = 16\pi\left[-\frac{1}{3\,(x-1)^3}\right]_{-3}^{-1} = \frac{7}{12}\,\pi$

$\displaystyle\pi\int_{-1}^{0}(-x)^2\,\mathrm{d}x = \pi\int_{-1}^{0}x^2\,\mathrm{d}x$

$\displaystyle = \pi\left[\frac{1}{3}\,x^3\right]_{-1}^{0} = \frac{1}{3}\,\pi$

$\displaystyle\Rightarrow \quad V = \frac{7}{12}\,\pi + \frac{1}{3}\,\pi = \boxed{\frac{11}{12}\,\pi\ \mathrm{u}^3}$

$\displaystyle\pi\int_{2}^{6} \frac{256}{(x+2)^4}\,\mathrm{d}x = 256\pi\int_{2}^{6}(x+2)^{-4}\,\mathrm{d}x$

$\displaystyle = 256\pi\left[-\frac{1}{3\,(x+2)^3}\right]_{2}^{6} = \frac{7}{6}\,\pi$

$\displaystyle\pi\int_{0}^{2}\left(\frac{1}{2}x\right)^2\,\mathrm{d}x = \pi\int_{0}^{2}\frac{1}{4}x^2\,\mathrm{d}x$

$\displaystyle = \pi\left[\frac{1}{12}\,x^3\right]_{0}^{2} = \frac{2}{3}\,\pi$

$\displaystyle\Rightarrow \quad V = \frac{7}{6}\,\pi + \frac{2}{3}\,\pi = \boxed{\frac{11}{6}\,\pi\ \mathrm{u}^3}$