Fonctions, équations, systèmes et inéquations de degré 1

Exercice 1

a) $(6x - 2) - (4 - x) = 0$

$6x - 2 - 4 + x = 0 \qquad \Rightarrow \qquad 7x = 6 \qquad \Rightarrow \qquad S = \left\{ \dfrac{6}{7} \right\}$

b) $585x - 702 - 135x = 65x - 260$

$385x = 442 \qquad \Rightarrow \qquad S = \left\{ \dfrac{442}{385} \right\}$

c) $140x + 80 - 54x - 72 - 5 = 25x + 125$

$61x = 122 \qquad \Rightarrow \qquad S = \{2\}$

d) $9x + 6 - 15x - (10x + 25) = 45$

$-6x + 6 - 10x - 25 = 45 \qquad \Rightarrow \qquad -16x = 64 \qquad \Rightarrow \qquad S = \{-4\}$

e) $3[12 + 4x - 5x + 4] - 3 + 6x$

$-3x + 48 - 3 + 6x = 0 \qquad \Rightarrow \qquad 3x = -45 \qquad \Rightarrow \qquad S = \{-15\}$

f) $35x - 30 - (100x - 10) = 3x - 24 - 200$

$35x - 30 - 100x + 10 = 3x - 224 \qquad \Rightarrow \qquad -68x = -204 \qquad \Rightarrow \qquad S = \{3\}$

Exercice 2

a) $\begin{cases} x + 2y = 4 \\ 5x - 2y = 8 \end{cases}$

$\overline{ 6x = 12 } \qquad \Rightarrow \qquad x = 2$

$\qquad \qquad \qquad \Rightarrow \qquad y = \dfrac{4 - 2}{2} = 1 \quad \Rightarrow \quad S = \{(2;\ 1)\}$

b) $\begin{cases} -2x - 4y = -2 \\ 2x + 5y = 3 \end{cases}$

$\overline{ y = 1 } \qquad \Rightarrow \quad x = 1 - 2 = -1 \quad \Rightarrow \quad S = \{(-1;\ 1)\}$

c)

$$\begin{cases} -20x + 28y = -108 \\ 20x + 30y = 50 \end{cases}$$

$$\overline{58y = -58} \quad \Rightarrow \quad y = -1$$

$$\Rightarrow \quad x = \frac{-7 + 27}{5} = 4 \quad \Rightarrow \quad S = \{(4;\ -1)\}$$

d)

$$\begin{cases} 3x + 2y = 21 \\ 2x - 2y = 4 \end{cases}$$

$$\overline{5x = 25} \quad \Rightarrow \quad x = 5$$

$$\Rightarrow \quad y = 5 - 2 = 3 \quad \Rightarrow \quad S = \{(5;\ 3)\}$$

e)

$$\begin{cases} 9x - 6y = 39 \\ 4x + 6y = 0 \end{cases}$$

$$\overline{13x = 39} \quad \Rightarrow \quad x = 3$$

$$\Rightarrow \quad y = \frac{-12}{6} = -2 \quad \Rightarrow \quad S = \{(3;\ -2)\}$$

f)

$$\begin{cases} 4x + 6y = 5 \\ -4x + 3y = -1 \end{cases}$$

$$\overline{9y = 4} \quad \Rightarrow \quad y = \frac{4}{9}$$

$$\Rightarrow \quad x = \frac{5 - \frac{8}{3}}{4} = \frac{7}{12} \quad \Rightarrow \quad S = \left\{ \left(\frac{7}{12};\ \frac{4}{9} \right) \right\}$$

Exercice 3

a) $x > -\dfrac{3}{7} \qquad \Rightarrow \qquad S = \left] -\dfrac{3}{7};\ +\infty \right[$

b) $x \leq \dfrac{9}{5} \qquad \Rightarrow \qquad S = \left] -\infty;\ \dfrac{9}{5} \right]$

c) $5 + 20x > 7 + 12x$

$8x > 2$

$x > \dfrac{1}{4} \qquad \Rightarrow \qquad S = \left] \dfrac{1}{4};\ +\infty \right[$

d) $-8x + 2x + 4 \leq -x$

$5x \geq 4 \qquad \Rightarrow \qquad S = \left[\dfrac{4}{5};\ +\infty \right[$

e) $15t - 4 < 2 + 12t$
   $3t < 6 \qquad \Rightarrow \qquad S = \,]{-\infty};\ 2[$

f) $4x - 20 - (3x - 24) \leq 0$
   $x \leq -4 \qquad \Rightarrow \qquad S = \,]{-\infty};\ -4]$

Exercice 4

a) ① $x < \dfrac{5}{2} \qquad \Rightarrow \qquad \left]{-\infty};\ \dfrac{5}{2}\right[$

   ② $x \geq -\dfrac{2}{3} \qquad \Rightarrow \qquad \left[-\dfrac{2}{3};\ +\infty\right[$

   $\Rightarrow \qquad S = \left[-\dfrac{2}{3};\ \dfrac{5}{2}\right[$

b) ① $x \geq \dfrac{13}{7} \qquad \Rightarrow \qquad \left[\dfrac{13}{7};\ +\infty\right[$

   ② $x > \dfrac{5}{3} \qquad \Rightarrow \qquad \left]\dfrac{5}{3};\ +\infty\right[$

   $\Rightarrow \qquad S = \left[\dfrac{13}{7};\ +\infty\right[$

c) ① $x < -\dfrac{7}{3} \qquad \Rightarrow \qquad \left]{-\infty};\ -\dfrac{7}{3}\right[$

   ② $x > -\dfrac{8}{3} \qquad \Rightarrow \qquad \left]-\dfrac{8}{3};\ +\infty\right[$

   $\Rightarrow \qquad S = \left]-\dfrac{8}{3};\ -\dfrac{7}{3}\right[$

d) ① $x \leq 2 \qquad \Rightarrow \qquad \,]{-\infty};\ 2]$

   ② $x \geq 2 \qquad \Rightarrow \qquad [2;\ +\infty[$

   $\Rightarrow \qquad S = \{2\}$

e) ① $x < -1 \qquad \Rightarrow \qquad \,]{-\infty};\ -1[$

   ② $x > 3 \qquad \Rightarrow \qquad \,]3;\ +\infty[$

   $\Rightarrow \qquad S = \emptyset$

f) ① $x < -2 \qquad \Rightarrow \qquad \,]{-\infty};\ -2[$

   ② $x < 1 \qquad \Rightarrow \qquad \,]{-\infty};\ 1[$

   ③ $x > -5 \qquad \Rightarrow \qquad \,]{-5};\ +\infty[$

   $\Rightarrow \qquad S = \,]{-5};\ -2[$

Exercice 5

a) $f(x) = mx + h$

$\begin{array}{l} f(3) = 1 \\ f(4) = -2 \end{array} \quad \Rightarrow \quad \left\{ \begin{array}{l} 3m + h = 1 \\ 4m + h = -2 \end{array} \right.$

$\left\{ \begin{array}{l} -3m - h = -1 \\ 4m + h = -2 \end{array} \right.$
$\overline{\qquad m = -3 \qquad} \quad \Rightarrow \qquad h = 1 + 9 = 10$
$\qquad\qquad\qquad\qquad \Rightarrow \qquad f(x) = -3x + 10$

b) $f(x) = mx + h$

$\begin{array}{l} f(1) = 4 \\ f(2) = 0 \end{array} \quad \Rightarrow \quad \left\{ \begin{array}{l} m + h = 4 \\ 2m + h = 0 \end{array} \right.$

$\left\{ \begin{array}{l} -m - h = -4 \\ 2m + h = 0 \end{array} \right.$
$\overline{\qquad m = -4 \qquad} \quad \Rightarrow \qquad h = 8$
$\qquad\qquad\qquad\qquad \Rightarrow \qquad f(x) = -4x + 8$

c) $f(x) = \frac{2}{3}x + 1$

d) $f(x) = -2x + h$
$\quad f(1) = 3 \qquad \Rightarrow \quad -2 + h = 3 \quad \Rightarrow \qquad h = 5 \quad \Rightarrow \quad f(x) = -2x + 5$

e) $f(x) = 7$

f) $f(x) = \frac{4}{5}x + h$
$\quad f(10) = -6 \qquad \Rightarrow \quad 8 + h = -6 \quad \Rightarrow \qquad h = -14 \quad \Rightarrow \quad f(x) = \frac{4}{5}x - 14$

Exercice 6

x = nombre de kilomètres parcourus
$f(x) = mx + h$

$\begin{array}{l} f(200) = 40 \\ f(450) = 15 \end{array} \quad \Rightarrow \quad \left\{ \begin{array}{l} 200m + h = 40 \\ 450m + h = 15 \end{array} \right.$

$\left\{ \begin{array}{l} -200m - h = -40 \\ 450m + h = 15 \end{array} \right.$
$\overline{\qquad 250m = -25 \qquad} \quad \Rightarrow \qquad m = -\frac{1}{10}$
$\qquad\qquad\qquad\qquad\quad \Rightarrow \qquad h = 40 + 20 = 60$
$\qquad\qquad\qquad\qquad\quad \Rightarrow \qquad f(x) = -\frac{1}{10}x + 60$

$f(0) = 60 \qquad \Rightarrow \qquad$ la capacité du réservoir est de 60 litres

consommation aux 100 km : $\dfrac{200}{20} = 10$ litres

Exercice 7

p = nombre de poules (avant l'intervention du chien)

d = nombre de dindons (avant l'intervention du chien)

$$\begin{cases} p + d = 36 \\ \dfrac{2}{5}p = \dfrac{1}{2}d \end{cases} \qquad \Rightarrow \qquad \begin{cases} p + d = 36 \\ 4p - 5d = 0 \end{cases}$$

$$\begin{array}{l} \begin{cases} 5p + 5d = 180 \\ 4p - 5d = 0 \end{cases} \\ \hline \quad 9p = 180 \end{array} \qquad \Rightarrow \qquad p = 20$$
$$\Rightarrow \qquad d = 36 - 20 = 16$$

Il y avait 20 poules et 16 dindons.

Exercice 8

c = nombre de cubes (12 arêtes)

p = nombre de pyramides (8 arêtes)

$$\begin{cases} c + p = 11 \\ 12c + 8p = 116 \end{cases}$$

$$\begin{array}{l} \begin{cases} -8c - 8p = -88 \\ 12c + 8p = 116 \end{cases} \\ \hline \quad 4c = 28 \end{array} \qquad \Rightarrow \qquad c = 7$$
$$\Rightarrow \qquad p = 11 - 7 = 4$$

Il y a 7 cubes et 4 pyramides.

Exercice 9

a = nombre d'ouvriers de l'usine A

b = nombre d'ouvriers de l'usine B

$$\begin{cases} a = 2b \\ \dfrac{1}{4}a + \dfrac{1}{5}b = 175 \end{cases} \qquad \Rightarrow \qquad \begin{cases} a - 2b = 0 \\ 5a + 4b = 3500 \end{cases}$$

$$\begin{cases} 2a - 4b = 0 \\ 5a + 4b = 3500 \end{cases}$$
$$\overline{7a = 3500} \qquad \Rightarrow \qquad a = 500$$
$$\qquad\qquad\qquad\qquad \Rightarrow \qquad b = \frac{500}{2} = 250$$

Il y a 500 ouvriers dans l'usine A et 250 dans l'usine B.

Exercice 10

x = nombre de pièces de fonte de 48 kg

y = nombre de pièces de fonte de 36 kg

$$\begin{cases} x + y = 80 \\ 48x + 36y = 3036 \end{cases}$$

$$\begin{cases} -36x - 36y = -2880 \\ 48x + 36y = 3036 \end{cases}$$
$$\overline{12x = 156} \qquad \Rightarrow \qquad x = 13$$
$$\qquad\qquad\qquad\qquad \Rightarrow \qquad y = 80 - 13 = 67$$

Il y a 13 pièces de fonte de 48 kg et 67 pièces de fonte de 36 kg.

Exercice 11

x = montant en francs de la première partie

y = montant en francs de la seconde partie

$$\begin{cases} x + y = 112000 \\ \dfrac{5x}{100} + \dfrac{3,5y}{100} = \dfrac{3,5x}{100} + \dfrac{5y}{100} + 420 \end{cases} \qquad \Rightarrow \qquad \begin{cases} x + y = 112000 \\ 1,5x - 1,5y = 42000 \end{cases}$$

$$\begin{cases} 1,5x + 1,5y = 168000 \\ 1,5x - 1,5y = 42000 \end{cases}$$
$$\overline{3x = 210000} \qquad \Rightarrow \qquad x = 70000$$
$$\qquad\qquad\qquad\qquad \Rightarrow \qquad y = 112000 - 70000 = 42000$$

La première partie vaut 70'000 francs et la deuxième 42'000 francs.

Exercice 12

e = prix au kg en francs du café Esquisito

d = prix au kg en francs du café Diavolo

$$\begin{cases} 24e + 32d = 896 \\ 32e + 24d = 868 \end{cases}$$

$$\begin{cases} 96e + 128d = 3584 \\ -96e - 72d = -2604 \end{cases}$$
$$\overline{ 56d = 980 } \quad \Rightarrow \quad d = 17,5$$
$$\Rightarrow \quad e = \frac{896 - 560}{24} = 14$$

Le prix au kg du café Esquisito vaut 14 francs et celui du café Diavolo vaut 17,50 francs.