

## Fonctions de degré 2

### Exercice 1

a)  $2x^2 + 5x + 2$

$$\Delta = 25 - 16 = 9 \quad \Rightarrow x = \frac{-5 \pm 3}{4} \quad \Rightarrow x = -\frac{1}{2} \text{ ou } x = -2$$

$$2x^2 + 5x + 2 = 2 \left( x + \frac{1}{2} \right) (x + 2) = (2x + 1)(x + 2)$$

b)  $6x^2 + 55x - 50$

$$\Delta = 3025 + 1200 = 4225 \quad \Rightarrow x = \frac{-55 \pm 65}{12} \quad \Rightarrow x = \frac{5}{6} \text{ ou } x = -10$$

$$6x^2 + 55x - 50 = 6 \left( x - \frac{5}{6} \right) (x + 10) = (6x - 5)(x + 10)$$

c)  $3x^2 - 13x + 14$

$$\Delta = 169 - 168 = 1 \quad \Rightarrow x = \frac{13 \pm 1}{6} \quad \Rightarrow x = \frac{7}{3} \text{ ou } x = 2$$

$$3x^2 - 13x + 14 = 3 \left( x - \frac{7}{3} \right) (x - 2) = (3x - 7)(x - 2)$$

d)  $4x^3 - 4x^2 + x = x(4x^2 - 4x + 1)$

$$\Delta = 16 - 16 = 0 \quad \Rightarrow x = \frac{4}{8} = \frac{1}{2}$$

$$4x^3 - 4x^2 + x = 4x \left( x - \frac{1}{2} \right)^2 = x(2x - 1)^2$$

### Exercice 2

a)  $3(x - 2) = (x - 2)^2$

$$3x - 6 = x^2 - 4x + 4$$

$$x^2 - 7x + 10 = 0 \quad \Rightarrow (x - 5)(x - 2) = 0 \quad \Rightarrow S = \{2; 5\}$$

b)  $4x^3 - 36x = 0$

$$4x(x^2 - 9) = 0 \quad \Rightarrow 4x(x - 3)(x + 3) = 0 \quad \Rightarrow S = \{-3; 0; 3\}$$

c)  $x^4 - 5x^2 + 4 = 0$

$$(x^2 - 4)(x^2 - 1) = 0 \quad \Rightarrow (x - 2)(x + 2)(x - 1)(x + 1) = 0 \quad \Rightarrow S = \{-2; -1; 1; 2\}$$

d)  $x^8 + 9x^4 - 10 = 0$

$$(x^4 - 1)(x^4 + 10) = 0 \quad \Rightarrow (x^2 - 1)(x^2 + 1)(x^4 + 10) = 0$$

$$(x - 1)(x + 1)(x^2 + 1)(x^4 + 10) = 0 \quad \Rightarrow S = \{-1; 1\}$$

e)  $(2x - 7)(5x + 3) = 8(4x^2 - 49)$

$$10x^2 - 29x - 21 = 32x^2 - 392 \quad \Rightarrow 22x^2 + 29x - 371 = 0$$

$$\Delta = 841 + 32648 = 33489 \quad \Rightarrow x = \frac{-29 \pm 183}{44} \quad \Rightarrow S = \left\{ -\frac{53}{11}; \frac{7}{2} \right\}$$

f)  $9x^2 - (2x + 5)^2 = 0$

$$9x^2 - (4x^2 + 20x + 25)^2 = 0 \quad \Rightarrow 5x^2 - 20x - 25 = 0$$

$$5(x^2 - 4x - 5) = 0 \quad \Rightarrow 5(x - 5)(x + 1) = 0 \quad \Rightarrow S = \{-1; 5\}$$

### Exercice 3

a)  $-x^2 + 3x + 4 > 0$

$$\Delta = 9 + 16 = 25 \quad \Rightarrow x = \frac{-3 \pm 5}{-2} \quad \Rightarrow x = -1 \text{ ou } x = 4$$

la parabole est concave  $\Rightarrow S = ] - 1; 4[$

b)  $4x^2 - 20x + 25 > 0$

$$\Delta = 400 - 400 = 0 \quad \Rightarrow x = \frac{20}{8} = \frac{5}{2}$$

la parabole est convexe  $\Rightarrow S = \mathbb{R} - \left\{ \frac{5}{2} \right\}$

c)  $3x^2 + 2 \leq 7 + 2x \quad \Rightarrow 3x^2 - 2x - 5 \leq 0$

$$\Delta = 4 + 60 = 64 \quad \Rightarrow x = \frac{2 \pm 8}{6} \quad \Rightarrow x = \frac{5}{3} \text{ ou } x = -1$$

la parabole est convexe  $\Rightarrow S = \left[ -1; \frac{5}{3} \right]$

d)  $(x - 2)^2 + 2x < -2 \quad \Rightarrow x^2 - 2x + 6 < 0$

$$\Delta = 4 - 24 = -20 < 0 \quad \Rightarrow \text{pas de zéro}$$

la parabole est convexe  $\Rightarrow S = \emptyset$

**Exercice 4**

a)  $f(x) = ax^2 + bx + c$

$$\begin{cases} \textcircled{1} & 4a + 2b + c = 9 \\ \textcircled{2} & 36a - 6b + c = -7 \\ \textcircled{3} & a + b + c = 0 \end{cases}$$

$$\begin{array}{r} -1 \cdot \textcircled{1} \\ 1 \cdot \textcircled{2} \end{array} \quad \begin{array}{l} -4a - 2b - c = -9 \\ \underline{36a - 6b + c = -7} \end{array}$$

$$32a - 8b = -16$$

$$\textcircled{4} \quad 4a - b = -2$$

$$\begin{array}{r} 1 \cdot \textcircled{2} \\ -1 \cdot \textcircled{3} \end{array} \quad \begin{array}{l} 36a - 6b + c = -7 \\ \underline{-a - b - c = 0} \end{array}$$

$$35a - 7b = -7$$

$$\textcircled{5} \quad 5a - b = -1$$

$$\begin{array}{r} -1 \cdot \textcircled{4} \\ 1 \cdot \textcircled{5} \end{array} \quad \begin{array}{l} -4a + b = 2 \\ \underline{5a - b = -1} \end{array}$$

$$a = 1 \quad \rightarrow \textcircled{4} \quad b = 4 + 2 = 6$$

$$\rightarrow \textcircled{3} \quad c = -1 - 6 = -7$$

$$\Rightarrow \boxed{f(x) = x^2 + 6x - 7}$$

b)  $f(x) = a(x - h)^2 + k$  avec  $S(h; k)$

$$\Rightarrow h = -1 \text{ et } k = -2 \quad \Rightarrow f(x) = a(x + 1)^2 - 2$$

$$f(2) = -3 \quad \Rightarrow a(2 + 1)^2 - 2 = -3 \quad \Rightarrow 9a - 2 = -3$$

$$\Rightarrow a = -\frac{1}{9} \quad \Rightarrow f(x) = -\frac{1}{9}(x + 1)^2 - 2$$

$$\Rightarrow \boxed{f(x) = -\frac{1}{9}x^2 - \frac{2}{9}x - \frac{19}{9}}$$

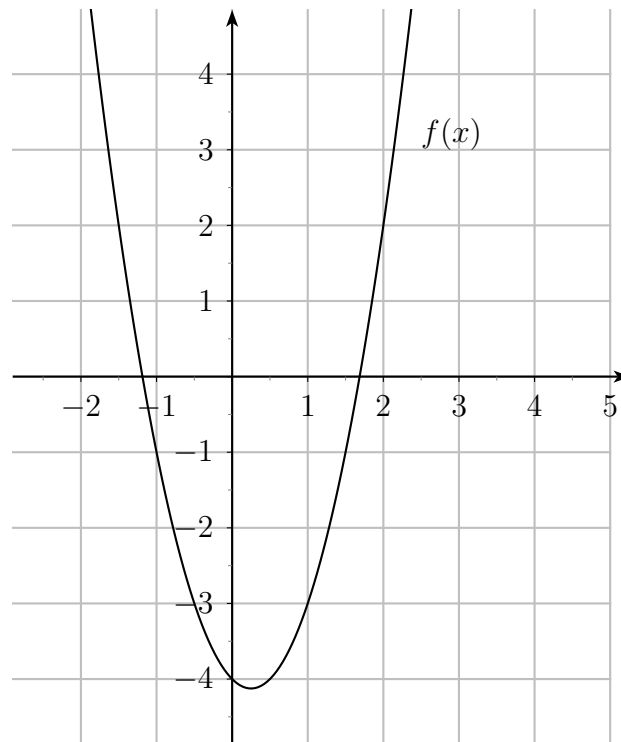
**Exercice 5**

axe de symétrie :  $\boxed{x = \frac{1}{4}}$

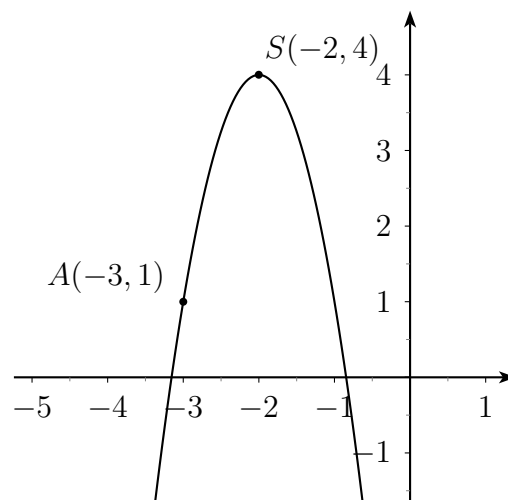
$$f\left(\frac{1}{4}\right) = \frac{1}{8} - \frac{1}{4} - 4 = -\frac{33}{8} \quad \Rightarrow \boxed{S = \left\{\frac{1}{4}; -\frac{33}{8}\right\}}$$

$$\Delta = 1 + 32 = 33 \quad \Rightarrow x = \frac{1 \pm \sqrt{33}}{4}$$

zéros de la fonction :  $\boxed{x = \frac{1 + \sqrt{33}}{4} \text{ ou } x = \frac{1 - \sqrt{33}}{4}}$



### Exercice 6



a)  $f(x) = a(x - h)^2 + k$  avec  $S(h; k)$   
 $\Rightarrow h = -2$  et  $k = 4$   $\Rightarrow f(x) = a(x + 2)^2 + 4$   
 $f(-3) = 1$   $\Rightarrow a(-3 + 2)^2 + 4 = 1$   $\Rightarrow a + 4 = 1$   
 $\Rightarrow a = -3$   $\Rightarrow f(x) = -3(x + 2)^2 + 4$   
 $\Rightarrow f(x) = -3x^2 - 12x - 8$

$$\begin{aligned} \text{b) } -3x^2 - 12x - 8 &= 4x^2 - 3x - 6 && \Rightarrow 7x^2 + 9x + 2 = 0 \\ \Delta = 81 - 56 = 25 & \Rightarrow x = \frac{-9 \pm 5}{14} && \Rightarrow x = -\frac{2}{7} \text{ ou } x = -1 \\ g\left(-\frac{2}{7}\right) &= \frac{16}{49} + \frac{6}{7} - 6 = -\frac{236}{49} \\ g(-1) &= 4 + 3 - 6 = 1 \\ \text{points d'intersection : } & \boxed{I_1\left(-\frac{2}{7}; -\frac{236}{49}\right) \text{ et } I_2(-1; 1)} \end{aligned}$$


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### Exercice 7

$x$  = largeur du cadre (en cm)

$$(2x + 84)(2x + 60) - 5040 = 5040 \quad \Rightarrow 4x^2 + 288x - 5040 = 0$$

$$x^2 + 72x - 1260 = 0$$

$$\Delta = 5184 + 5040 = 10224 \quad \Rightarrow x = \frac{-72 \pm \sqrt{10224}}{2}$$

$$\Rightarrow x = \frac{-72 + \sqrt{10224}}{2} \cong 14,56 \text{ ou } x = \frac{-72 - \sqrt{10224}}{2} \cong -86,56 < 0 \text{ (sol. à éliminer)}$$

La largeur du cadre vaut  $\sim 14,56$  cm.

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### Exercice 8

$x$  = âge actuel du fils

$x + 24$  = âge actuel du père

$$x(x + 24) = 3x^2 \quad \Rightarrow 2x^2 - 24x = 0$$

$$2x(x - 12) = 0 \quad \Rightarrow x = 12 \text{ ou } x = 0 \text{ (sol. à éliminer)}$$

Le fils a 12 ans et le père 36 ans.