Trigonométrie

Exercice 1

a) $AB = 48,5 \cdot sin(5,16^\circ) \cong \boxed{\text{4,36 m}}$

b) $\sphericalangle AHB = arcsin\left(\frac{4,76}{48,5}\right) \cong \boxed{5{,}63^\circ}$

Exercice 2

a) $\boxed{150^\circ}$ b) $\boxed{171{,}89^\circ}$ c) $\boxed{-54^\circ}$ d) $\boxed{45{,}84^\circ}$

1) $\boxed{\frac{2\pi}{5}}$ 2) $\boxed{\frac{\pi}{12}}$ 3) $\boxed{-\frac{3\pi}{10}}$ 4) $\boxed{\frac{5\pi}{3}}$

Exercice 3

$10^\circ 15' = 10{,}25^\circ$

distance entre les deux villes : $6370 \cdot 10,25 \cdot \frac{\pi}{180} \cong \boxed{\text{1139,57 km}}$

Exercice 4

Résoudre les équations.

a) $arcsin\left(-\frac{\sqrt{3}}{2}\right) = -60^\circ$ $x = -60^\circ$ ou $180^\circ - (-60^\circ)$

$\boxed{x_1 = -60^\circ + k \cdot 360^\circ \quad \text{avec } k \in \mathbb{Z}}$

$\boxed{x_2 = 240^\circ + k \cdot 360^\circ \quad \text{avec } k \in \mathbb{Z}}$

b) $arccos\left(\frac{3}{4}\right) \cong 41,41^\circ$ $5x = \pm 41,41^\circ$

$\boxed{x_1 \cong 8,28^\circ + k \cdot 72^\circ \quad \text{avec } k \in \mathbb{Z}}$

$\boxed{x_2 \cong -8,28^\circ + k \cdot 72^\circ \quad \text{avec } k \in \mathbb{Z}}$

c) $arccos(0,2) \cong 78,46^\circ$ $\quad t + 90^\circ = \pm 78,46^\circ$

$\boxed{t_1 \cong -11,54^\circ + k \cdot 360^\circ \quad \text{avec } k \in \mathbb{Z}}$

$\boxed{t_2 \cong -168,46^\circ + k \cdot 360^\circ \quad \text{avec } k \in \mathbb{Z}}$

d) $cos^2(x) = 1 - sin^2(x)$ $\quad \Rightarrow 3(1 - sin^2(x)) + sin(x) - 1 = 0$

$3 - 3sin^2(x) + sin(x) - 1 = 0$

$3sin^2(x) - sin(x) - 2 = 0$

$(3sin(x) + 2)(sin(x) - 1) = 0 \quad \Rightarrow sin(x) = -\frac{2}{3}$ ou $sin(x) = 1$

$arcsin\left(-\frac{2}{3}\right) \cong -41,81^\circ \quad x = -41,81^\circ$ ou $180^\circ - (-41,81^\circ)$

$\boxed{x_1 = -41,81^\circ + k \cdot 360^\circ \quad \text{avec } k \in \mathbb{Z}}$

$\boxed{x_2 = 221,41^\circ + k \cdot 360^\circ \quad \text{avec } k \in \mathbb{Z}}$

$arcsin(1) = 90^\circ \quad x = 90^\circ$ ($180^\circ - 90^\circ = 90^\circ$, 2e solution identique)

$\boxed{x_3 = 90^\circ + k \cdot 360^\circ \quad \text{avec } k \in \mathbb{Z}}$

Exercice 5

$\sphericalangle BAD = 180^\circ - 90^\circ - 50^\circ = 40^\circ \quad \Rightarrow \sphericalangle CAD = 30^\circ + 40^\circ = 70^\circ$

théorème du sinus dans le triangle ACD : $\quad \dfrac{42}{sin(70^\circ)} = \dfrac{AC}{sin(50^\circ)}$

$$AC = \frac{42 \cdot sin(50^\circ)}{sin(70^\circ)} \cong 34{,}24 \text{ m}$$

$AB = AC \cdot cos(30^\circ) \cong \boxed{29{,}65 \text{ m}}$

Exercice 6

théorème du cosinus : $\quad \alpha = arccos\left(\dfrac{9^2 + 4^2 - 6^2}{2 \cdot 9 \cdot 4}\right) \cong \boxed{32{,}09^\circ}$

théorème du sinus : $\quad \dfrac{6}{sin(\alpha)} = \dfrac{9}{sin(\beta)} = \underbrace{\dfrac{4}{sin(\gamma)}}_{\text{angle aigu}}$

$$\gamma = arcsin\left(\frac{4 \cdot sin(\alpha)}{6}\right) \cong \boxed{20{,}74^\circ}$$

$\beta = 180^\circ - \alpha - \gamma \cong \boxed{127{,}17^\circ}$

$S = \sigma(\Delta\ ABC) = \frac{1}{2} \cdot 9 \cdot 4 \cdot sin(\alpha) \cong \boxed{9{,}56\ u^2}$

Exercice 7

théorème du sinus : $$\frac{5}{sin(35^\circ)} = \frac{7}{\underbrace{sin(\beta)}_{\text{angle aigu ou obtus}}} = \frac{c}{sin(\gamma)}$$

$\beta_1 = arcsin\left(\frac{7 \cdot sin(35^\circ)}{5}\right) \cong \boxed{53{,}42^\circ}$	$\beta_2 = 180^\circ - \beta_1 \cong \boxed{126{,}58^\circ}$
$\gamma_1 = 180^\circ - 35^\circ - \beta_1 \cong \boxed{91{,}58^\circ}$	$\gamma_2 = 180^\circ - 35^\circ - \beta_2 \cong \boxed{18{,}42^\circ}$
$c_1 = \frac{5 \cdot sin(\gamma_1)}{sin(35^\circ)} \cong \boxed{8{,}71\ u}$	$c_2 = \frac{5 \cdot sin(\gamma_2)}{sin(35^\circ)} \cong \boxed{2{,}75\ u}$
$S_1 = \sigma(\Delta ABC) = \frac{1}{2} \cdot 5 \cdot 7 \cdot sin(\gamma_1) \cong \boxed{17{,}49\ u^2}$	$S_2 = \sigma(\Delta ABC) = \frac{1}{2} \cdot 5 \cdot 7 \cdot sin(\gamma_2) \cong \boxed{5{,}53\ u^2}$

Exercice 8

$\sphericalangle\ ABS = 180^\circ - 60^\circ = 120^\circ \qquad \Rightarrow \sphericalangle ASB = 180^\circ - 40^\circ - 120^\circ = 20^\circ$

théorème du sinus dans le triangle ABS : $$\frac{300}{sin(20^\circ)} = \frac{BS}{sin(40^\circ)}$$

$BS = \frac{300 \cdot sin(40^\circ)}{sin(20^\circ)} \cong 563{,}82$ m

$h = BS \cdot sin(60^\circ) \cong 488{,}28$ m

altitude du sommet S : $1250 + h \cong \boxed{1738{,}28 \text{ m}}$