Géométrie Analytique

Exercice 1

a) $\overrightarrow{d} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \quad \Rightarrow \quad \overrightarrow{n} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad \Rightarrow \quad (a):\ 2x + 4y + c = 0$

$A \in a \quad \Rightarrow \quad -12 + 12 + c = 0 \quad \Rightarrow c = 0 \quad \Rightarrow \quad \boxed{(a):\ x + 2y = 0}$

b) $(b):\ y = -\frac{1}{5}x + h \qquad B \in b \quad \Rightarrow \quad 2 = 1 + h \quad \Rightarrow h = 1$

$\Rightarrow y = -\frac{1}{5}x + 1 \quad \Rightarrow \quad \boxed{(b):\ x + 5y - 5 = 0}$

c) $\overrightarrow{n} = \begin{pmatrix} 6 \\ -1 \end{pmatrix} \quad \Rightarrow \quad (c):\ 6x - y + c = 0$

$C \in c \quad \Rightarrow \quad 36 + 1 + c = 0 \quad \Rightarrow c = -37 \quad \Rightarrow \quad \boxed{(c):\ 6x - y - 37 = 0}$

d) $\overrightarrow{d} = \begin{pmatrix} \frac{3}{2} \\ -\frac{5}{3} \end{pmatrix} \quad \Rightarrow \quad \overrightarrow{n} = \begin{pmatrix} \frac{5}{3} \\ \frac{3}{2} \end{pmatrix} \quad \Rightarrow \quad (d):\ \frac{5}{3}x + \frac{3}{2}y + c = 0$

$D \in d \quad \Rightarrow \quad \frac{10}{9} - \frac{1}{4} + c = 0 \quad \Rightarrow c = -\frac{31}{36} \quad \Rightarrow \quad \boxed{(d):\ 60x + 54y - 31 = 0}$

e) $(e):\ 3x - 2y + c = 0 \qquad E \in e \quad \Rightarrow \quad 6 - 8 + c = 0 \quad \Rightarrow c = 2$

$\Rightarrow \boxed{(e):\ 3x - 2y + 2 = 0}$

f) $m = \frac{6}{-12} = -\frac{1}{2} \quad \Rightarrow \quad (f):\ y = -\frac{1}{2}x + h$

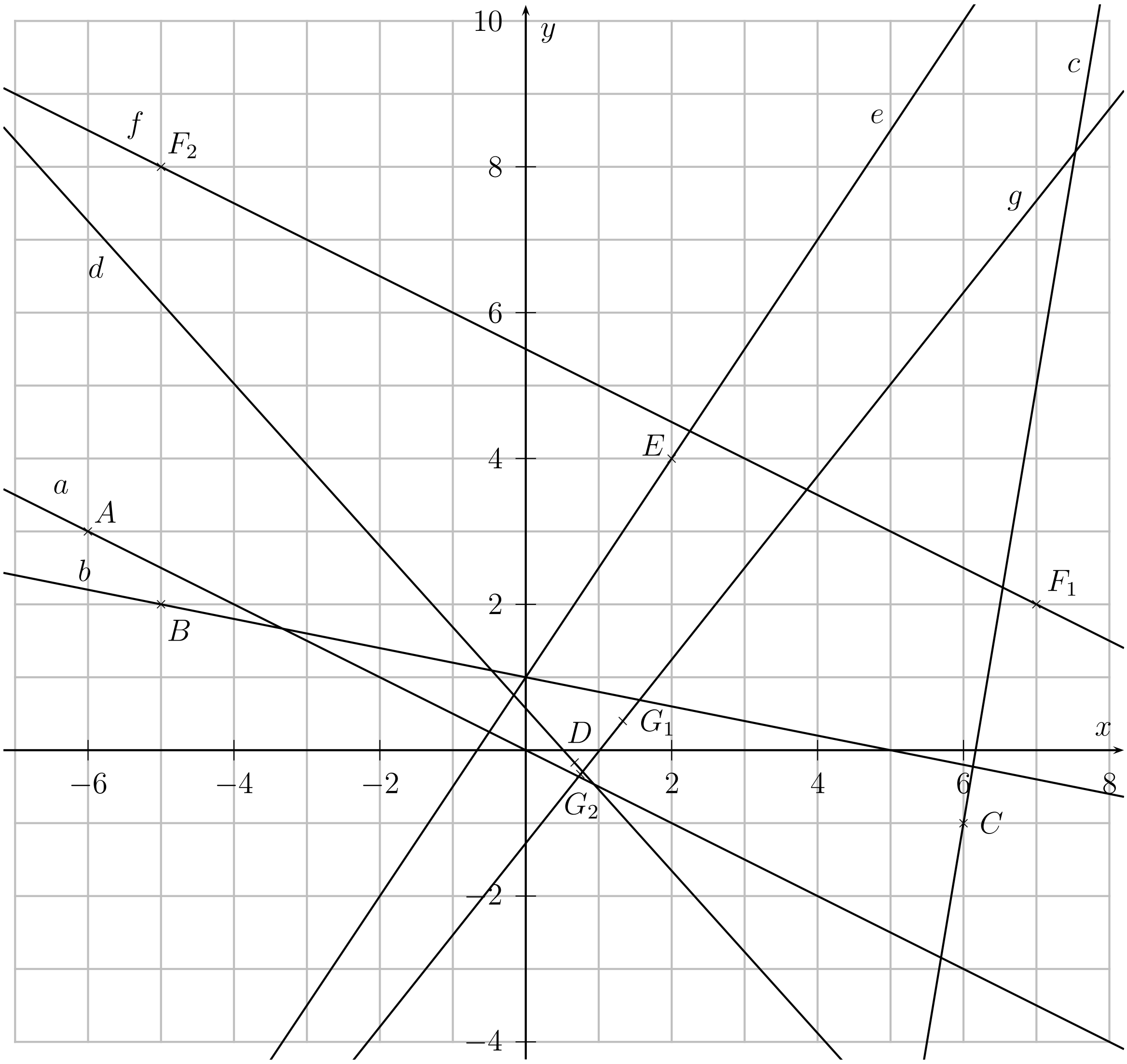
$F_1 \in f \quad \Rightarrow \quad 2 = -\frac{7}{2} + h \quad \Rightarrow h = \frac{11}{2} \quad \Rightarrow y = -\frac{1}{2}x + \frac{11}{2}$

$\Rightarrow \boxed{(f):\ x + 2y - 11 = 0}$

g) $m = \dfrac{-\frac{11}{15}}{-\frac{7}{12}} = \frac{44}{35} \quad \Rightarrow \quad (g):\ y = \frac{44}{35}x + h$

$G_1 \in g \quad \Rightarrow \quad \frac{2}{5} = \frac{176}{105} + h \quad \Rightarrow h = -\frac{134}{105} \quad \Rightarrow y = \frac{44}{35}x - \frac{134}{105}$

$\Rightarrow \boxed{(g):\ 132x - 105y - 134 = 0}$



Exercice 2

intersection de a et d :

$-12 - 15m + 9 + 5m - 7 = 0 \quad \Rightarrow \quad -10m = 10 \quad \Rightarrow \quad m = -1 \quad \Rightarrow \quad A(1;4)$

intersection de a et b :

$3x + \frac{5}{4}x - \frac{3}{2} - 7 = 0 \quad \Rightarrow \quad 17x = 34 \quad \Rightarrow \quad x = 2 \quad \Rightarrow \quad y = 1 \Rightarrow \quad B(2;1)$

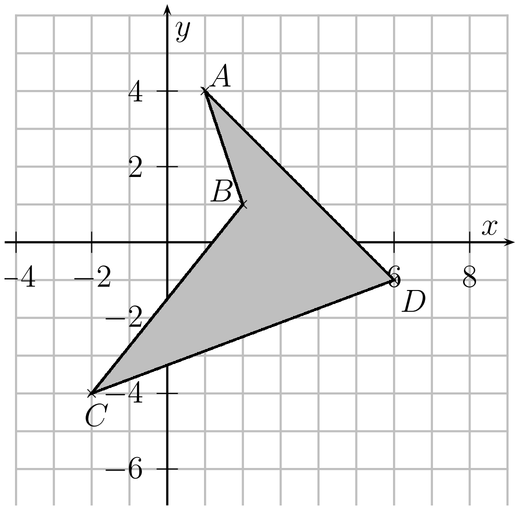
intersection de b et c :

$2 + 3k = \frac{35}{2} + 10k - \frac{3}{2} \quad \Rightarrow \quad -7k = 14 \quad \Rightarrow \quad k = -2 \quad \Rightarrow \quad C(-2;-4)$

intersection de c et d :

$$\begin{cases} 14 + 8k = -4 - 5m \\ 2 + 3k = 9 + 5m \end{cases} \quad \Rightarrow \quad \begin{cases} 8k + 5m = -18 \\ 3k - 5m = 7 \end{cases} \quad \Rightarrow \quad 11k = -11 \quad \Rightarrow \quad k = -1$$

$\Rightarrow m = -2 \quad \Rightarrow \quad D(6;-1)$



$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 2-1 \\ 1-4 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\overrightarrow{BD} = \overrightarrow{OD} - \overrightarrow{OB} = \begin{pmatrix} 6-2 \\ -1-1 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

aire du $\Delta\ ABD$: $\frac{1}{2} \cdot |\det(\overrightarrow{AB}; \overrightarrow{BD})| = \frac{1}{2} \cdot |1 \cdot (-2) - (-3) \cdot 4| = 5\ u^2$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} -2-2 \\ -4-1 \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$

aire du $\Delta\ BCD$: $\frac{1}{2} \cdot |\det(\overrightarrow{BC}; \overrightarrow{BD})| = \frac{1}{2} \cdot |(-4) \cdot (-2) - (-5) \cdot 4| = 14\ u^2$

aire du quadrilatère $ABCD$: $5 + 14 = \boxed{19\ u^2}$

Exercice 3

a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} -3-7 \\ 2-5 \end{pmatrix} = \begin{pmatrix} -10 \\ -3 \end{pmatrix}$

$$\Rightarrow \boxed{(c) : \begin{cases} x = 7 - 10k \\ y = 5 - 3k \end{cases} \quad \text{avec } k \in \mathbb{R}}$$

b) milieu du segment AC : $N\left(\frac{7+8}{2}; \frac{5-4}{2}\right) \quad \Rightarrow \quad N\left(\frac{15}{2}; \frac{1}{2}\right)$

$(m_B) : \overrightarrow{OB} + k \cdot \overrightarrow{BN} \qquad \overrightarrow{BN} = \overrightarrow{ON} - \overrightarrow{OB} = \begin{pmatrix} \frac{15}{2} + 3 \\ \frac{1}{2} - 2 \end{pmatrix} = \begin{pmatrix} \frac{21}{2} \\ -\frac{3}{2} \end{pmatrix}$

$$(m_B) : \begin{cases} x = -3 + \frac{21}{2}k \\ y = 2 - \frac{3}{2}k \end{cases} \quad \text{avec } k \in \mathbb{R}$$

$$\begin{cases} x = -3 + \frac{21}{2}k \\ 7y = 14 - \frac{21}{2}k \end{cases} \qquad x + 7y = 11 \quad \Rightarrow \boxed{(m_B) : x + 7y - 11 = 0}$$

c) droite (BC) : $\overrightarrow{OB} + k \cdot \overrightarrow{BC}$ $\qquad \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} 8+3 \\ -4-2 \end{pmatrix} = \begin{pmatrix} 11 \\ -6 \end{pmatrix}$

$$(a) : \begin{cases} x = -3 + 11k \\ y = 2 - 6k \end{cases} \qquad \text{avec } k \in \mathbb{R}$$

$$\begin{cases} 6x = -18 + 66k \\ 11y = 22 - 66k \end{cases} \qquad 6x + 11y = 4$$

$$(a) : 6x + 11y - 4 = 0 \quad \Rightarrow \quad \vec{n} = \begin{pmatrix} 6 \\ 11 \end{pmatrix}$$

$$\overrightarrow{AB} \cdot \vec{n} = -60 - 33 = -93 \quad \text{et} \quad \|\vec{n}\| = \sqrt{36 + 121} = \sqrt{157}$$

$$\delta(A; a) = \frac{|-93|}{\sqrt{157}} = \frac{93}{\sqrt{157}} = \boxed{\frac{93\sqrt{157}}{157}\, u}$$

d) aire du $\Delta\ ABC$: $\frac{1}{2} \cdot |\det(\overrightarrow{AB}; \overrightarrow{BC})| = \frac{1}{2} \cdot |(-10) \cdot (-6) - (-3) \cdot 11| = \boxed{\frac{93}{2}\, u^2}$

e) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 8-7 \\ -4-5 \end{pmatrix} = \begin{pmatrix} 1 \\ -9 \end{pmatrix}$

$$\overrightarrow{AB'} = \frac{\overrightarrow{AC} \cdot \overrightarrow{AB}}{\|\overrightarrow{AC}\|^2} \cdot \overrightarrow{AC} = \frac{-10 + 27}{82} \cdot \begin{pmatrix} 1 \\ -9 \end{pmatrix} = \boxed{\tfrac{17}{82} \begin{pmatrix} 1 \\ -9 \end{pmatrix}}$$

f) $\overrightarrow{OB'} = \overrightarrow{OA} + \overrightarrow{AB'} = \begin{pmatrix} 7 + \frac{17}{82} \\ 5 - \frac{153}{82} \end{pmatrix} = \begin{pmatrix} \frac{591}{82} \\ \frac{257}{82} \end{pmatrix}$

$$\Rightarrow \boxed{B'\left(\frac{591}{82}; \frac{257}{82}\right)}$$

g) $G\left(\frac{7-3+8}{3}; \frac{5+2-4}{3}\right) \quad \Rightarrow \boxed{G(4; 1)}$

h) droite $(a) : 6x + 11y - 4 = 0$

$(d) : 6x + 11y + c = 0$

$A \in d : 42 + 55 + c = 0 \qquad c = -97 \quad \Rightarrow \quad (d) :\ 6x + 11y - 97 = 0$

$$\Rightarrow \boxed{(d) : y = -\tfrac{6}{11}x + \tfrac{97}{11}}$$