Intégrales

Exercice 1

a) $\displaystyle\int (4x+3)\,dx = \boxed{2x^2+3x+c}$ avec $c \in \mathbb{R}$

b) $\displaystyle\int \left(\frac{1}{z^3}-\frac{3}{z^2}\right)\,dz = \int z^{-3}\,dz - 3\int z^{-2}\,dz = -\frac{1}{2}z^{-2} - 3\cdot(-1)z^{-1}+c$

$\displaystyle = \boxed{-\frac{1}{2z^2}+\frac{3}{z}+c}$ avec $c \in \mathbb{R}$

c) $\displaystyle\int \left(2x^{\frac{5}{4}}+6x^{\frac{1}{4}}+3x^{-4}\right)\,dx = 2\cdot\frac{4}{9}x^{\frac{9}{4}}+6\cdot\frac{4}{5}x^{\frac{5}{4}}+3\cdot\left(-\frac{1}{3}\right)x^{-3}+c$

$\displaystyle = \boxed{\frac{8}{9}\sqrt[4]{x^9}+\frac{24}{5}\sqrt[4]{x^5}-\frac{1}{x^3}+c}$ avec $c \in \mathbb{R}$

d) $\displaystyle\int x\,(2x+3)\,dx = \int (2x^2+3x)\,dx = \boxed{\frac{2}{3}x^3+\frac{3}{2}x^2+c}$ avec $c \in \mathbb{R}$

e) $\displaystyle\int \left(\sqrt{t}+cos(t)\right)\,dt = \int t^{\frac{1}{2}}\,dt + \int cos(t)\,dt = \frac{2}{3}t^{\frac{3}{2}}+sin(t)+c$

$\displaystyle = \boxed{\frac{2}{3}\sqrt{t^3}+sin(t)+c}$ avec $c \in \mathbb{R}$

f) $\displaystyle\int x\,(2x^2+3)^{10}\,dx = \frac{1}{4}\int \underbrace{4x}_{u'(x)}\,\underbrace{(2x^2+3)^{10}}_{u^{10}(x)}\,dx = \frac{1}{4}\cdot\frac{1}{11}(2x^2+3)^{11}+c$

$\displaystyle = \boxed{\frac{1}{44}(2x^2+3)^{11}+c}$ avec $c \in \mathbb{R}$

g) $\displaystyle\int \frac{x}{\sqrt[3]{1-2x^2}}\,dx = -\frac{1}{4}\int \underbrace{-4x}_{u'(x)}\,\underbrace{(1-2x^2)^{-\frac{1}{3}}}_{u^{-\frac{1}{3}}(x)}\,dx = -\frac{1}{4}\cdot\frac{3}{2}(1-2x^2)^{\frac{2}{3}}+c$

$\displaystyle = \boxed{-\frac{3}{8}\sqrt[3]{(1-2x^2)^2}+c}$ avec $c \in \mathbb{R}$

h) $\displaystyle\int \frac{x^2+x}{(4-3x^2-2x^3)^4}\,dx = -\frac{1}{6}\int \underbrace{(-6x^2-6x)}_{u'(x)}\,\underbrace{(4-3x^2-2x^3)^{-4}}_{u^{-4}(x)}\,dx$

$\displaystyle = -\frac{1}{6}\cdot\left(-\frac{1}{3}\right)(4-3x^2-2x^3)^{-3}+c = \boxed{\frac{1}{18\,(4-3x^2-2x^3)^3}+c}$ avec $c \in \mathbb{R}$

Exercice 2

$$\begin{cases} a(t) = a = \text{cte} \\ v(t) = at + v_0 \\ s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \end{cases}$$

$a = -g = -9,81\ m/s^2 \qquad v_0 = 500\ m/s \qquad s_0 = 0\ m$

$$\Rightarrow \begin{cases} v(t) = -gt + 500 \\ s(t) = -\frac{1}{2}gt^2 + 500t \end{cases}$$

a) $\boxed{s(t) = -\frac{1}{2}gt^2 + 500t}$

b) le projectile touche le sol lorsque $s(t) = 0$

$\Rightarrow -\frac{1}{2}gt^2 + 500t = 0 \qquad \Rightarrow -\frac{1}{2}t(gt - 1000) = 0$

$\Rightarrow t = 0 \qquad \text{ou} \qquad t = \frac{1000}{g} \cong 101.94$

la trajectoire est une parabole (concave), donc le maximum est situé entre les deux zéros losque $t = \frac{500}{g} \cong 50,97\,s$

$$h_{\max} = s\left(\frac{500}{g}\right) = -\frac{125000}{g} + \frac{250000}{g} = \boxed{\frac{125000}{g} \cong 12'742,1\ m}$$

Exercice 3

$$\begin{cases} a(t) = a = \text{cte} \\ v(t) = at + v_0 \\ s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \end{cases}$$

$v_0 = 0\ m/s \qquad s_0 = 0\ m$

$$\Rightarrow \begin{cases} v(t) = at \\ s(t) = \frac{1}{2}at^2 \end{cases}$$

$s(10) = 150 \qquad \Rightarrow 50a = 150 \qquad \boxed{\Rightarrow a = 3\ m/s^2}$

Exercice 4

$$\begin{cases} a(t) = a = \text{cte} \\ v(t) = at + v_0 \\ s(t) = \dfrac{1}{2}at^2 + v_0 t + s_0 \end{cases}$$

$$v_0 = 60\ km/h = \frac{50}{3}\ m/s = 16,\overline{6}\ m/s \qquad s_0 = 0\ m$$

$$\Rightarrow \begin{cases} v(t) = at + \dfrac{50}{3} \\ \\ s(t) = \dfrac{1}{2}at^2 + \dfrac{50}{3}t \end{cases}$$

$$v(9) = 0 \qquad \Rightarrow 9a + \frac{50}{3} = 0 \qquad \boxed{\Rightarrow a = -\frac{50}{27} = -1,\overline{851}\ m/s^2}$$

Exercice 5

$$\begin{cases} a(t) = a = \text{cte} \\ v(t) = at + v_0 \\ s(t) = \dfrac{1}{2}at^2 + v_0 t + s_0 \end{cases}$$

$$a = -g \cong -9,81\ m/s^2 \qquad v_0 = -9\ m/s \qquad s_0 = 275\ m$$

$$\Rightarrow \begin{cases} v(t) = -gt - 9 \\ s(t) = -\dfrac{1}{2}gt^2 - 9t + 275 \end{cases}$$

a) $\boxed{s(t) = -\dfrac{1}{2}gt^2 - 9t + 275}$

b) $\boxed{v(5) = -5g - 9 \cong -58,05\ m/s}$

c) $s(t) = 0 \qquad \Rightarrow -\dfrac{1}{2}gt^2 - 9t + 275 = 0$

$\Rightarrow gt^2 + 18t - 550 = 0 \qquad \Delta = 324 + 2200g \cong 21906$

$$\Rightarrow t = \frac{-18 \pm \sqrt{21906}}{2g} \cong \begin{cases} 6,63 \\ -8,46\ \text{(sol. à éliminer)} \end{cases}$$

$\boxed{\text{la pierre touche le sol après environ 6,63 secondes}}$

Exercice 6

a) $$\int_1^3 (x^2+1)\,dx = \left[\frac{1}{3}x^3 + x\right]_1^3 = 9+3-\frac{1}{3}-1 = \boxed{\frac{32}{3}\ u^2}$$

b) $$\int_1^4 \frac{1}{x^3}\,dx = \int_1^4 x^{-3}\,dx = \left[-\frac{1}{2}x^{-2}\right]_1^4 = \left[-\frac{1}{2x^2}\right]_1^4 = -\frac{1}{32}+\frac{1}{2} = \boxed{\frac{15}{32}\ u^2}$$

c) $$\int_1^1 3x^2\sqrt{x^3+x}\,dx = \boxed{0\ u^2}$$ (les bornes sont identiques !)

d) $$\int_0^2 x^2\sqrt{x^3+1}\,dx = \frac{1}{3}\int_0^2 \underbrace{3x^2}_{u'(x)}\underbrace{(x^3+1)^{\frac{1}{2}}}_{u^{\frac{1}{2}}(x)}\,dx = \left[\frac{1}{3}\cdot\frac{2}{3}(x^3+1)^{\frac{3}{2}}\right]_0^2 = \left[\frac{2}{9}\sqrt{(x^3+1)^3}\right]_0^2$$

$$= 6-\frac{2}{9} = \boxed{\frac{52}{9}\ u^2}$$

e) $$\int_0^1 (2x-3)(5x+1)\,dx = \int_0^1 (10x^2-13x-3)\,dx = \left[\frac{10}{3}x^3-\frac{13}{2}x^2-3x\right]_0^1$$

$$= \frac{10}{3}-\frac{13}{2}-3-0 = \boxed{-\frac{37}{6}\ u^2}$$

f) $$\int_0^1 \sqrt[3]{8x^7}\,dx = 2\int_0^1 x^{\frac{7}{3}}\,dx = \left[2\cdot\frac{3}{10}\cdot x^{\frac{10}{3}}\right]_0^1 = \left[\frac{3}{5}\cdot\sqrt[3]{x^{10}}\right]_0^1 = \frac{3}{5}-0 = \boxed{\frac{3}{5}\ u^2}$$

g) $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cos\left(\tfrac{1}{3}x\right)\,dx = 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underbrace{\frac{1}{3}}_{u'(x)}\underbrace{cos\left(\frac{1}{3}x\right)}_{cos(u(x))} = \left[3\cdot sin\left(\tfrac{1}{3}x\right)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{3}{2}+\frac{3}{2} = \boxed{3\ u^2}$$

h) $$\int_0^{\frac{\pi}{3}} \frac{sin(x)}{cos^2(x)}\,dx = -\int_0^{\frac{\pi}{3}} \underbrace{-sin(x)}_{u'(x)}\underbrace{cos^{-2}(x)}_{u^{-2}(x)}\,dx = \left[-(-1)cos^{-1}(x)\right]_0^{\frac{\pi}{3}} = \left[\frac{1}{cos(x)}\right]_0^{\frac{\pi}{3}}$$

$$= 2-1 = \boxed{1\ u^2}$$