Exercice 2.6.7

$x^2 - px + 36 = 0$

$\Delta = p^2 - 144 = (p - 12)(p + 12)$

$\Delta > 0$ si $m \in]-\infty; -12[\cup]12; +\infty[$

a) $\Delta = 0 \quad \Rightarrow \quad p = \pm 12$

b) $\Delta > 0$ et $x' = -x''$

$\quad \Rightarrow \quad x' + x'' = -\dfrac{b}{a} = p = 0$

si $p = 0 \quad \Rightarrow \quad \Delta < 0$, donc impossible

c) $\Delta > 0$ et $x'^2 + x''^2 = 184 \qquad x' + x'' = p \qquad x' \cdot x'' = 36$

$\quad (x' + x'')^2 = x'^2 + 2x'x'' + x''^2 \quad \Leftrightarrow \quad p^2 = 184 + 72 \quad \Leftrightarrow \quad p^2 = 256$

$\quad \Leftrightarrow \quad p = \pm 16$

d) $\Delta > 0$ et $\dfrac{1}{x'} + \dfrac{1}{x''} = \dfrac{5}{12} \quad \Leftrightarrow \quad \dfrac{x' + x''}{x'x''} = \dfrac{5}{12} \quad \Leftrightarrow \quad \dfrac{p}{36} = \dfrac{5}{12} \quad \Leftrightarrow \quad p = 15$

Exercice 2.6.8

$x^2 - (m - 2)x + 2m - 7 = 0$

$\Delta = (m - 2)^2 - 4(2m - 7) = m^2 - 4m + 4 - 8m + 28 = m^2 - 12m + 32 = (m - 4)(m - 8)$

$\Delta > 0$ si $m \in]-\infty; 4[\cup]8; +\infty[$

a) $\Delta > 0$ et $x' + x'' = -\dfrac{b}{a} = m - 2 = 0 \quad \Leftrightarrow \quad m = 2$

b) $\Delta > 0$ et $x' + x'' > 0$ et $x'x'' > 0$

$\quad \Delta > 0 \quad \Leftrightarrow \quad m \in]-\infty; 4[\cup]8; +\infty[$

$\quad x' + x'' > 0 \quad \Leftrightarrow \quad m - 2 > 0 \quad \Leftrightarrow \quad m > 2$

$\quad x'x'' > 0 \quad \Leftrightarrow \quad 2m - 7 > 0 \quad \Leftrightarrow \quad m > \dfrac{7}{2}$

$\quad \Rightarrow \quad m \in \left]\frac{7}{2}; 4\right[\cup]8; +\infty[$

c) 1 solution "double", $\Delta = 0 \quad \Rightarrow m_1 = 4$ et $m_2 = 8$

Exercice 2.7.1

$(5x+5)^2 = (5x+3)^2 + (2x+4)^2 \quad \Leftrightarrow \quad 25x^2 + 50x + 25 = 25x^2 + 30x + 9 + 4x^2 + 16x + 16$

$\Leftrightarrow \quad 4x^2 - 4x = 0 \quad \Leftrightarrow \quad 4x(x-1) = 0$

si $x = 0$ les dimensions sont 3 - 4 - 5

si $x = 1$ les dimensions sont 6 - 8 - 10