# Équations et systèmes

**Exercice 1.**

$ED = \mathbb{R} - \{-1; 1\}$

$2(x-1) + 2x - 3 - 2(x^2 - 1) = -(x+1)$

$2x - 2 + 2x - 3 - 2x^2 + 2 = -x - 1$

$2x^2 - 5x + 2 = 0 \quad \Rightarrow \quad (2x-1)(x-2) = 0$

$\Rightarrow \boxed{\mathcal{S} = \left\{\dfrac{1}{2}; 2\right\}}$

$ED = \mathbb{R} - \{-1; 1\}$

$-3(x+1) + x - 5 + 2(x^2 - 1) = x - 1$

$-3x - 3 + x - 5 + 2x^2 - 2 = x - 1$

$2x^2 - 3x - 9 = 0 \quad \Rightarrow \quad (2x+3)(x-3) = 0$

$\Rightarrow \boxed{\mathcal{S} = \left\{-\dfrac{3}{2}; 3\right\}}$

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**Exercice 2.**

$3 - x \geq 0 \quad \Rightarrow \quad x \leq 3$

$\Rightarrow \quad ED = ]-\infty; 3]$

$\sqrt{3-x} = x - 1 \qquad x - 1 \geq 0$

$\Rightarrow \quad x \geq 1 \quad \Rightarrow \quad ED' = [1; 3]$

$3 - x = (x-1)^2 \quad \Rightarrow \quad 3 - x = x^2 - 2x + 1$

$x^2 - x - 2 = 0 \quad \Rightarrow \quad (x-2)(x+1) = 0$

$\Rightarrow \boxed{\mathcal{S} = \{2\}}$ (sol. $-1$ à élim.)

$4 - x \geq 0 \quad \Rightarrow \quad x \leq 4$

$\Rightarrow \quad ED = ]-\infty; 4]$

$\sqrt{4-x} = -x + 4 \qquad -x + 4 \geq 0$

$\Rightarrow \quad x \leq 4 \quad \Rightarrow \quad ED' = ED$

$4 - x = (-x+4)^2 \quad \Rightarrow \quad 4 - x = x^2 - 8x + 16$

$x^2 - 7x + 12 = 0 \quad \Rightarrow \quad (x-4)(x-3) = 0$

$\Rightarrow \boxed{\mathcal{S} = \{3; 4\}}$

**Exercice 3.**

$$\Delta = (m+1)^2 - 4(4m-11)$$

$$= m^2 + 2m + 1 - 16m + 44$$

$$= m^2 - 14m + 45 = (m-5)(m-9)$$

2 sol. si $m \in ]-\infty;5[\cup]9;+\infty[$

a) $-\dfrac{b}{a} = \dfrac{m+1}{1} = 0 \quad \Leftrightarrow \quad \boxed{m = -1}$

b) $\dfrac{c}{a} = \dfrac{4m-11}{1} = 1 \quad \Leftrightarrow \quad \boxed{m = 3}$

c) $\dfrac{c}{a} = \dfrac{4m-11}{1} = 0 \quad \Leftrightarrow \quad \boxed{m = \dfrac{11}{4}}$

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$$\Delta = (m+2)^2 - 4(4m-8)$$

$$= m^2 + 4m + 4 - 16m + 32$$

$$= m^2 - 12m + 36 = (m-6)^2$$

2 sol. si $m \in \mathbb{R} - \{6\}$

$-\dfrac{b}{a} = \dfrac{m+2}{1} = 0 \quad \Leftrightarrow \quad \boxed{m = -2}$

$\dfrac{c}{a} = \dfrac{4m-8}{1} = 1 \quad \Leftrightarrow \quad \boxed{m = \dfrac{9}{4}}$

$\dfrac{c}{a} = \dfrac{4m-8}{1} = 0 \quad \Leftrightarrow \quad \boxed{m = 2}$

**Exercice 4.**

a) $y = \frac{5x+3}{4}$

$x^2 - \frac{(5x+3)^2}{16} + 2x \cdot \frac{5x+3}{4} + 3x + \frac{5x+3}{4} - 6 = 0$

$16x^2 - (5x+3)^2 + 8x(5x+3) + 48x + 4(5x+3) - 96 = 0$

$31\,x^2 + 62\,x - 93 = 0 \quad \Rightarrow \quad x^2 + 2\,x - 3 = 0$

$(x+3)(x-1) = 0 \quad \Rightarrow \quad x_1 = -3 \quad x_2 = 1$

$\Rightarrow \quad y_1 = -3 \quad y_2 = 2$

$\Rightarrow \boxed{\mathcal{S} = \{(-3;-3);(1;2)\}}$

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$y = \frac{7x+5}{4}$

$x^2 - \frac{(7x+5)^2}{16} + 2x \cdot \frac{7x+5}{4} + 3x + \frac{7x+5}{4} - 4 = 0$

$16x^2 - (7x+5)^2 + 8x(7x+5) + 48x + 4(7x+5) - 64 = 0$

$23\,x^2 + 46\,x - 69 = 0 \quad \Rightarrow \quad x^2 + 2\,x - 3 = 0$

$(x+3)(x-1) = 0 \quad \Rightarrow \quad x_1 = -3 \quad x_2 = 1$

$\Rightarrow \quad y_1 = -4 \quad y_2 = 3$

$\Rightarrow \boxed{\mathcal{S} = \{(-3;-4);(1;3)\}}$

b) $\begin{cases} -7x & + & 3y & - & z & = & 28 \\ 9x & + & 3y & + & 3z & = & -24 \\ -26x & + & 8y & - & 2z & = & 86 \end{cases}$

$\begin{cases} -7x & + & 3y & - & z & = & 28 \\ 3x & + & y & + & z & = & -8 \\ -13x & + & 4y & - & z & = & 43 \end{cases}$

$\begin{cases} -4x & + & 4y & = & 20 \\ -10x & + & 5y & = & 35 \end{cases}$

$\begin{cases} -x & + & y & = & 5 \\ -2x & + & y & = & 7 \end{cases} \quad \Rightarrow \quad x = -2$

$\Rightarrow \quad y = 3 \quad \Rightarrow \quad z = -5 \quad \Rightarrow \quad t = 8$

$\Rightarrow \boxed{\mathcal{S} = \{(-2; 3; -5; 8)\}}$

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$\begin{cases} -7x & + & 3y & - & z & = & -29 \\ 9x & + & 3y & + & 3z & = & 27 \\ -26x & + & 8y & - & 2z & = & -88 \end{cases}$

$\begin{cases} -7x & + & 3y & - & z & = & -29 \\ 3x & + & y & + & z & = & 9 \\ -13x & + & 4y & - & z & = & -44 \end{cases}$

$\begin{cases} -4x & + & 4y & = & -20 \\ -10x & + & 5y & = & -35 \end{cases}$

$\begin{cases} -x & + & y & = & -5 \\ -2x & + & y & = & -7 \end{cases} \quad \Rightarrow \quad x = 2$

$\Rightarrow \quad y = -3 \quad \Rightarrow \quad z = 6 \quad \Rightarrow \quad t = -7$

$\Rightarrow \boxed{\mathcal{S} = \{(2; -3; 6; -7)\}}$

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## Exercice 5.

$u$ = chiffre des unités / cent. de milliers
$d$ = chiffre des dizaines / diz. de milliers
$c$ = chiffre des centaines / milliers

$\begin{cases} c + d + u = 17 \\ d = 3c \\ 10d + u - (10u + d) = 36 \end{cases}$

$\begin{cases} 4c + u = 17 \\ 27c - 9u = 36 \end{cases} \quad \Rightarrow \quad \begin{cases} 4c + u = 17 \\ 3c - u = 4 \end{cases}$

$\Rightarrow \quad 7c = 21 \quad \Rightarrow \quad c = 3 \quad \Rightarrow \quad d = 9$

$\Rightarrow \quad u = 5 \quad \Rightarrow \boxed{\text{le nombre est } 593'395}$

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$u$ = chiffre unités / cent. de milliers
$d$ = chiffre des dizaines / diz. de milliers
$c$ = chiffre des centaines / milliers

$\begin{cases} c + d + u = 11 \\ c = 2u \\ 10d + u - (10u + d) = 27 \end{cases}$

$\begin{cases} d + 3u = 11 \\ 9d - 9u = 27 \end{cases} \quad \Rightarrow \quad \begin{cases} d + 3u = 11 \\ d - u = 3 \end{cases}$

$\Rightarrow \quad 4u = 8 \quad \Rightarrow \quad u = 2 \quad \Rightarrow \quad c = 4$

$\Rightarrow \quad d = 5 \quad \Rightarrow \boxed{\text{le nombre est } 254'452}$