Fonctions

Exercice 1.

a) $f(x) = ax^2 + bx + c$

$$\begin{matrix}①\\②\\③\end{matrix}\left\{\begin{array}{c} a+b+c=0\\ 4a-2b+c=3\\ a-b+c=-2\end{array}\right.$$

$$\begin{array}{rl} 1\cdot② & 4a-2b+c=3\\ -1\cdot① & -a-b-c=0\\ \hline ④ & 3a-3b=3\end{array}$$

$$\begin{array}{rl} 1\cdot① & a+b+c=0\\ -1\cdot③ & -a+b-c=2\\ \hline ⑤ & 2b=2\end{array}$$

$\Rightarrow b=1 \quad \to ④ \quad a=2 \quad \to ① \quad c=-3$

$\Rightarrow \boxed{f(x)=2x^2+x-3}$

$f(x) = ax^2 + bx + c$

$$\begin{matrix}①\\②\\③\end{matrix}\left\{\begin{array}{c} a+b+c=-2\\ 4a+2b+c=3\\ a-b+c=6\end{array}\right.$$

$$\begin{array}{rl} 1\cdot② & 4a+2b+c=3\\ -1\cdot① & -a-b-c=2\\ \hline ④ & 3a+b=5\end{array}$$

$$\begin{array}{rl} 1\cdot① & a+b+c=-2\\ -1\cdot③ & -a+b-c=-6\\ \hline ⑤ & 2b=-8\end{array}$$

$\Rightarrow b=-4 \quad \to ④ \quad a=3 \quad \to ① \quad c=-1$

$\Rightarrow \boxed{f(x)=3x^2-4x-1}$

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b) $m=\dfrac{\Delta y}{\Delta x}=\dfrac{4}{(-1)}=-4$

$g(x)=-4x+h$

$1=-8+h \quad \Rightarrow \quad h=9$

$\Rightarrow \boxed{g(x)=-4x+9}$

$m=\dfrac{\Delta y}{\Delta x}=\dfrac{3}{1}=3$

$g(x)=3x+h$

$2=-3+h \quad \Rightarrow \quad h=5$

$\Rightarrow \boxed{g(x)=3x+5}$

$2x^2+x-3 \geq -4x+9 \quad \Leftrightarrow \quad 2x^2+5x-12 \geq 0$

$\Leftrightarrow \quad (2x-3)(x+4) \geq 0$

$\Rightarrow \boxed{\mathcal{S}=]-\infty;-4] \cup [\tfrac{3}{2};+\infty[}$

$3x^2-4x-1 \geq 3x+5 \quad \Leftrightarrow \quad 3x^2-7x-6 \geq 0$

$\Leftrightarrow \quad (3x+2)(x-3) \geq 0$

$\Rightarrow \boxed{\mathcal{S}=]-\infty;-\tfrac{2}{3}] \cup [3;+\infty[}$

Exercice 2.

$f(x) = \dfrac{(x-1)^2(2x-1)}{(x-5)(x+4)}$

$ED(f) = \mathbb{R} - \{-4; 5\}$

x	$-\infty$	-4		$\frac{1}{2}$		1		5	$+\infty$
$f(x)$	$-$	‖	$+$	0	$-$	0	$-$	‖	$+$

$f(x) = \dfrac{(x-1)(3x+1)(x-2)}{(x-4)(x+2)}$

$ED(f) = \mathbb{R} - \{-2; 4\}$

x	$-\infty$	-2		$-\frac{1}{3}$		1		2		4	$+\infty$
$f(x)$	$-$	‖	$+$	0	$-$	0	$+$	0	$-$	‖	$+$

Exercice 3.

a) $ED = \mathbb{R} - \{-3; 3\}$

$$\frac{5x^2+2}{x^2-9} > \frac{(5x-4)(x+3)}{x^2-9}$$

$$\Rightarrow \frac{5x^2+2-5x^2-11x+12}{x^2-9} > 0$$

$$\Rightarrow \frac{-11x+14}{x^2-9} > 0$$

x	$-\infty$	-3		$\frac{14}{11}$		3	$+\infty$
$\cdots$	$+$	‖	$-$	0	$+$	‖	$-$

$\Rightarrow \boxed{\mathcal{S} =]-\infty; -3[\cup]\frac{14}{11}; 3[}$

$ED = \mathbb{R}^* - \{5\}$

$$\frac{2x^2+1}{x^2-5x} > \frac{(2x+1)x}{x^2-5x}$$

$$\Rightarrow \frac{2x^2+1-2x^2-x}{x^2-5x} > 0$$

$$\Rightarrow \frac{-x+1}{x^2-5x} > 0$$

x	$-\infty$	0		1		5	$+\infty$
$\cdots$	$+$	‖	$-$	0	$+$	‖	$-$

$\Rightarrow \boxed{\mathcal{S} =]-\infty; 0[\cup]1; 5[}$

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b) $ED = \mathbb{R} - \{-4; 4\}$

$$\Rightarrow \frac{x(x-7)(x+2)}{(x-4)(x+4)} \leq 0$$

x	$-\infty$	-4		-2		0		4		7	$+\infty$
$f(x)$	$-$	‖	$+$	0	$-$	0	$+$	‖	$-$	0	$+$

$\Rightarrow \boxed{\mathcal{S} =]-\infty; -4[\cup [-2; 0] \cup]4; 7]}$

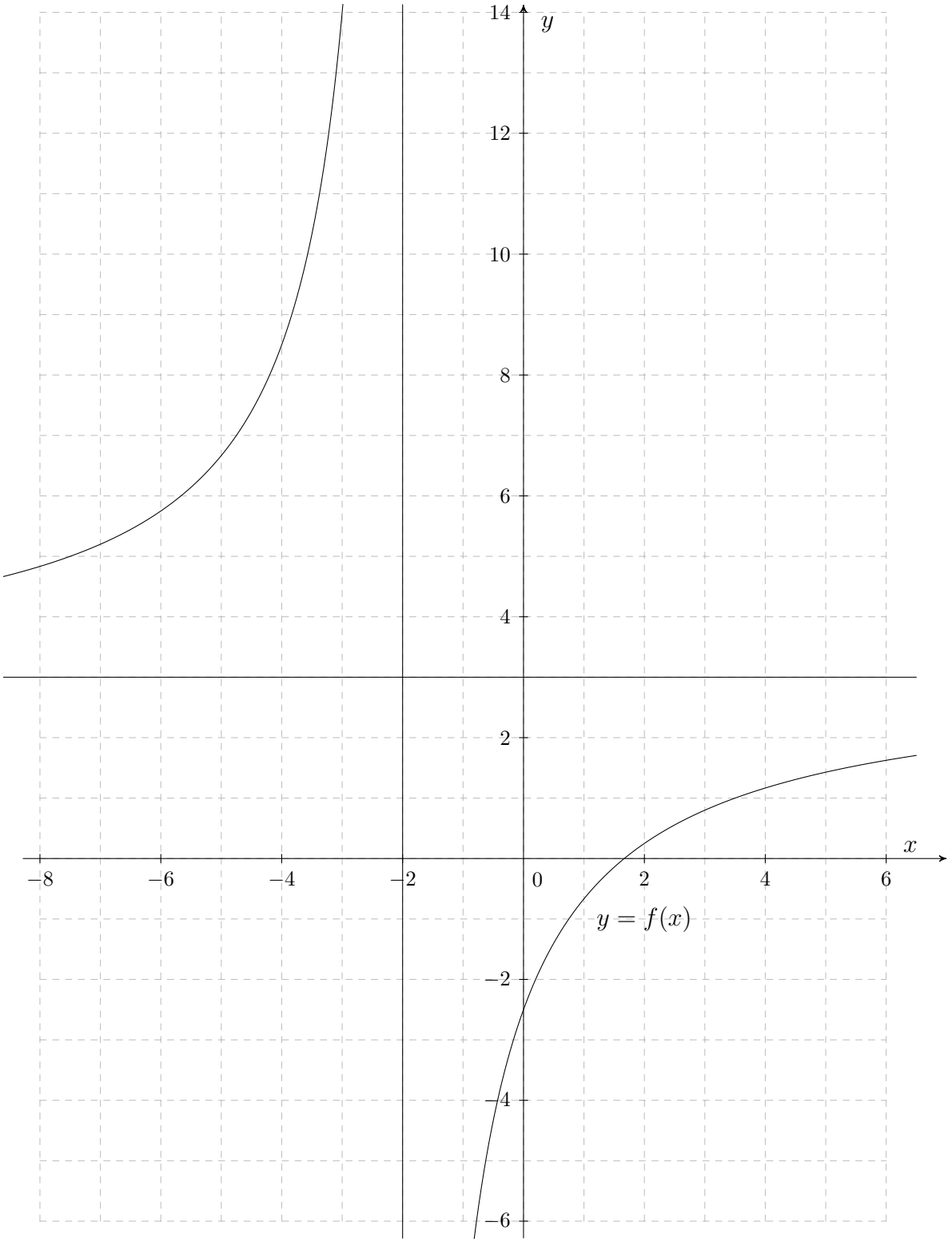
$ED = \mathbb{R} - \{-6; 6\}$

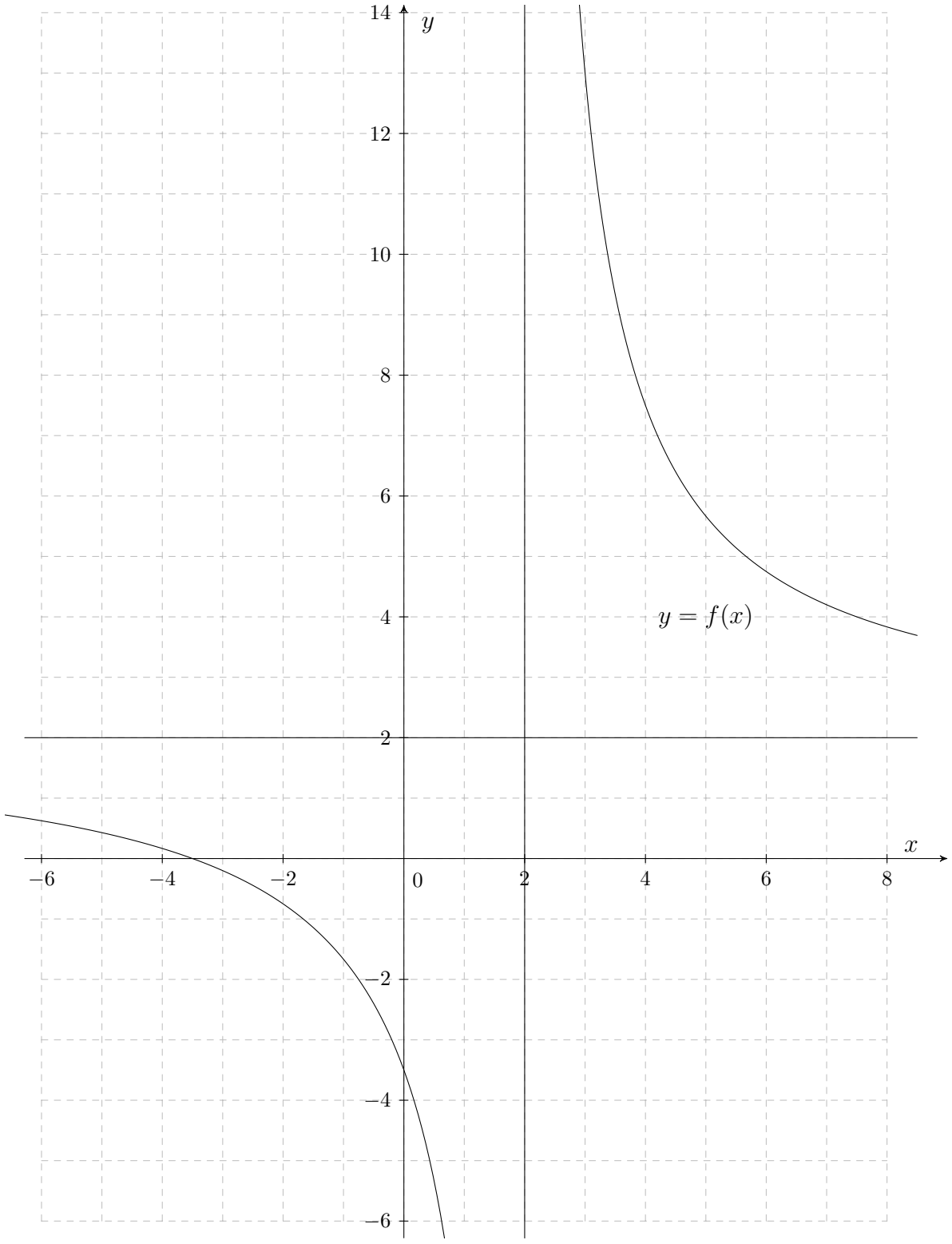
$$\Rightarrow \frac{x(x-3)(x-5)}{(x-6)(x+6)} \leq 0$$

x	$-\infty$	-6		0		3		5		6	$+\infty$
$f(x)$	$-$	‖	$+$	0	$-$	0	$+$	0	$-$	‖	$+$

$\Rightarrow \boxed{S =]-\infty; -6[\cup [0; 3] \cup [5; 6[}$

Exercice 4.




y
x
$y = f(x)$
14
12
10
8
6
4
2
−2
−4
−6
−6
−4
−2
0
2
4
6
8