Trigonométrie

Exercice 1.

$\alpha = 48,25°$

$\beta = \frac{670}{6370} \cdot \frac{180}{\pi} \simeq 6,026° = 6°\ 01'$

$\Rightarrow$ latitude de $B = \alpha - \beta \simeq \boxed{42°14'\text{ N}}$

$\alpha = 44,25°$

$\beta = \frac{850}{6370} \cdot \frac{180}{\pi} \simeq 7,645° = 7°\ 39'$

$\Rightarrow$ latitude de $B = \alpha - \beta \simeq \boxed{36°36'\text{ N}}$

Exercice 2.

a) $\widehat{ADC} = 180° - 30° - 40° = 110°$

thm du sin : $\frac{100}{\sin(110°)} = \frac{AD}{\sin(40°)}$

$\Leftrightarrow \quad AD = \frac{100 \cdot \sin(40°)}{\sin(110°)} \simeq 68.4$ m

thm du cos :
$BD^2 = 50^2 + 68.4^2 - 2 \cdot 50 \cdot 68.4 \cdot \cos(30°)$

$\Leftrightarrow \quad BD \simeq \boxed{35.43\text{ m}}$

b) thm du sin : $\frac{BD}{\sin(30°)} = \frac{50}{\sin(\widehat{ADB})}$

$\Leftrightarrow \quad \sin(\widehat{ADB}) = \frac{50 \cdot \sin(30°)}{BD} \simeq 0.71$

$\Rightarrow \quad \widehat{ADB} \simeq 44.88°$

$\Rightarrow \alpha = 180° - 90° - 30° - \widehat{ADB} \simeq \boxed{15.12°}$

$\widehat{ADC} = 180° - 30° - 50° = 100°$

thm du sin : $\frac{120}{\sin(100°)} = \frac{AD}{\sin(50°)}$

$\Leftrightarrow \quad AD = \frac{120 \cdot \sin(50°)}{\sin(100°)} \simeq 93.34$ m

thm du cos :
$BD^2 = 60^2 + 93.34^2 - 2 \cdot 60 \cdot 93.34 \cdot \cos(30°)$

$\Leftrightarrow \quad BD \simeq \boxed{51.11\text{ m}}$

thm du sin : $\frac{BD}{\sin(30°)} = \frac{60}{\sin(\widehat{ADB})}$

$\Leftrightarrow \quad \sin(\widehat{ADB}) = \frac{60 \cdot \sin(30°)}{BD} \simeq 0.59$

$\Rightarrow \quad \widehat{ADB} \simeq 35.94°$

$\Rightarrow \alpha = 180° - 90° - 30° - \widehat{ADB} \simeq \boxed{24.06°}$

Exercice 3.

$$\sin(\alpha) = \frac{h_1}{d}$$

$$\Leftrightarrow \quad h_1 = \sin(\alpha) \cdot d = 6.18$$

$$\cos(\alpha) = \frac{x}{d}$$

$$\Leftrightarrow \quad x = \cos(\alpha) \cdot d = 19.02$$

$$\tan(\theta) = \frac{h_2}{x}$$

$$\Leftrightarrow \quad h_2 = \tan(\theta) \cdot x = 13.42$$

$$\Rightarrow \quad T = h_2 - (h_1 - 1.75) = \boxed{8.99 \text{ m}}$$

$$\omega = 90^\circ - \theta = \boxed{54{,}8^\circ}$$

$$\sin(\alpha) = \frac{h_1}{d}$$

$$\Leftrightarrow \quad h_1 = \sin(\alpha) \cdot d = 3.88$$

$$\cos(\alpha) = \frac{x}{d}$$

$$\Leftrightarrow \quad x = \cos(\alpha) \cdot d = 14.49$$

$$\tan(\theta) = \frac{h_2}{x}$$

$$\Leftrightarrow \quad h_2 = \tan(\theta) \cdot x = 8.84$$

$$\Rightarrow \quad T = h_2 - (h_1 - 1.8) = \boxed{6.76 \text{ m}}$$

$$\omega = 90^\circ - \theta = \boxed{58.6^\circ}$$

Exercice 4.

$$\text{thm du sinus : } \frac{b}{\sin(\beta)} = 2R$$

$$\Leftrightarrow \sin(\beta) = \frac{b}{2R} = \frac{3}{5}$$

$$\Rightarrow \quad \beta = \arcsin\left(\frac{3}{5}\right) \simeq 36.87^\circ$$

$$\text{thm du sinus : } \frac{a}{\sin(\alpha)} = 2R$$

$$\Leftrightarrow \sin(\alpha) = \frac{a}{2R} = \frac{7}{10}$$

$$\Rightarrow \quad \alpha_1 = \arcsin\left(\frac{7}{10}\right) \simeq 44.43^\circ$$

$$\text{et } \alpha_2 = 180^\circ - \alpha_1 \simeq 135.57^\circ$$

$$\Rightarrow \gamma_1 = 180^\circ - \alpha_1 - \beta \simeq 98.7^\circ$$

$$\text{et } \gamma_2 = 180^\circ - \alpha_2 - \beta \simeq 7.56^\circ$$

$$\text{thm du sinus : } \frac{b}{\sin(\beta)} = 2R$$

$$\Leftrightarrow \sin(\beta) = \frac{b}{2R} = \frac{5}{8}$$

$$\Rightarrow \quad \beta = \arcsin\left(\frac{5}{8}\right) \simeq 38.68^\circ$$

$$\text{thm du sinus : } \frac{a}{\sin(\alpha)} = 2R$$

$$\Leftrightarrow \sin(\alpha) = \frac{a}{2R} = \frac{7}{8}$$

$$\Rightarrow \quad \alpha_1 = \arcsin\left(\frac{7}{10}\right) \simeq 61.04^\circ$$

$$\text{et } \alpha_2 = 180^\circ - \alpha_1 \simeq 118.96^\circ$$

$$\Rightarrow \gamma_1 = 180^\circ - \alpha_1 - \beta \simeq 80.27^\circ$$

$$\text{et } \gamma_2 = 180^\circ - \alpha_2 - \beta \simeq 22.36^\circ$$

thm du sinus : $\dfrac{c}{\sin(\gamma)} = 2R$

$\Leftrightarrow c_1 = 2R \cdot \sin(\gamma_1) \simeq 9.88$ cm

et $c_2 = 2R \cdot \sin(\gamma_2) \simeq 1.32$ cm

aire du $\Delta\ ABC = \dfrac{a \cdot b \cdot \sin(\gamma)}{2}$

$\Rightarrow \quad \Delta_1 \simeq 20.76$ cm^2 et $\Delta_2 \simeq 2.76$ cm^2

thm du sinus : $\dfrac{c}{\sin(\gamma)} = 2R$

$\Leftrightarrow c_1 = 2R \cdot \sin(\gamma_1) \simeq 7.88$ cm

et $c_2 = 2R \cdot \sin(\gamma_2) \simeq 3.04$ cm

aire du $\Delta\ ABC = \dfrac{a \cdot b \cdot \sin(\gamma)}{2}$

$\Rightarrow \quad \Delta_1 \simeq 17.25$ cm^2 et $\Delta_2 \simeq 6.66$ cm^2

Exercice 5.

a) $\dfrac{\sin(t)}{\cos(t)} = \dfrac{1}{\cos(t)} - 2\cos(t)$

$\Rightarrow \quad \sin(t) = 1 - 2\cos^2(t)\ (\cos(t) \neq 0)$

$\Rightarrow \quad \sin(t) = 1 - 2 + 2\sin^2(t)$

$\Leftrightarrow \quad 2\sin^2(t) - \sin(t) - 1 = 0$

$x = \sin(t) \quad \Rightarrow \quad 2x^2 - x - 1 = 0$

$\Leftrightarrow \quad (2x+1)(x-1) = 0$

$\Rightarrow \quad x = \sin(t) = -\dfrac{1}{2}$ ($x = 1$ sol à élim.)

$\arcsin\left(-\dfrac{1}{2}\right) = -30°$

$$\Rightarrow \quad \boxed{\begin{cases} t_1 = -30° + k \cdot 360° \quad \text{avec} \quad k \in \mathbb{Z} \\ t_2 = 210° + k \cdot 360° \quad \text{avec} \quad k \in \mathbb{Z} \end{cases}}$$

$\dfrac{\sin(x)}{\cos(x)} + 2\cos(x) = \dfrac{1}{\cos(x)}$

$\Rightarrow \quad \sin(x) + 2\cos^2(x) = 1\ (\cos(t) \neq 0)$

$\Rightarrow \quad \sin(x) + 2 - 2\sin^2(x) = 1$

$\Leftrightarrow \quad 2\sin^2(x) - \sin(x) - 1 = 0$

$y = \sin(x) \quad \Rightarrow \quad 2y^2 - y - 1 = 0$

$\Leftrightarrow \quad (2y+1)(y-1) = 0$

$\Rightarrow \quad y = \sin(x) = -\dfrac{1}{2}$ ($y = 1$ sol à élim.)

$\arcsin\left(-\dfrac{1}{2}\right) = -30°$

$$\Rightarrow \quad \boxed{\begin{cases} x_1 = -30° + k \cdot 360° \quad \text{avec} \quad k \in \mathbb{Z} \\ x_2 = 210° + k \cdot 360° \quad \text{avec} \quad k \in \mathbb{Z} \end{cases}}$$

b) $\dfrac{\sin(x)}{\cos(x)} = -3 \quad \Rightarrow \quad \tan(x) = -3$

$\arctan(-3) \simeq -71,56°$

$\Rightarrow \quad \boxed{x \simeq -71.56° + k \cdot 180° \quad \text{avec} \quad k \in \mathbb{Z}}$

$\dfrac{\sin(t)}{\cos(t)} = \dfrac{1}{4} \quad \Rightarrow \quad \tan(x) = \dfrac{1}{4}$

$\arctan\left(\dfrac{1}{4}\right) \simeq 14.04°$

$\Rightarrow \quad \boxed{x \simeq 14.04° + k \cdot 180° \quad \text{avec} \quad k \in \mathbb{Z}}$

Exercice 6.

$$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\Rightarrow \begin{cases} 3x_1 = \dfrac{\pi}{6} + k \cdot 2\pi \\ 3x_2 = \dfrac{5\pi}{6} + k \cdot 2\pi \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} x_1 = \dfrac{\pi}{18} + k \cdot \dfrac{2}{3}\pi \quad \text{avec} \quad k \in \mathbb{Z} \\ x_2 = \dfrac{5\pi}{18} + k \cdot \dfrac{2}{3}\pi \quad \text{avec} \quad k \in \mathbb{Z} \end{cases}}$$

$$\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

$$\Rightarrow \begin{cases} 4x_1 = \dfrac{\pi}{3} + k \cdot 2\pi \\ 4x_2 = \dfrac{2\pi}{3} + k \cdot 2\pi \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} x_1 = \dfrac{\pi}{12} + k \cdot \dfrac{1}{2}\pi \quad \text{avec} \quad k \in \mathbb{Z} \\ x_2 = \dfrac{\pi}{6} + k \cdot \dfrac{1}{2}\pi \quad \text{avec} \quad k \in \mathbb{Z} \end{cases}}$$