

Factorisation et équations

Exercice 1.

$$\text{a) } 1 \left| \begin{array}{ccc|c} 1 & 0 & -7 & 6 \\ & 1 & 1 & -6 \\ 1 & 1 & -6 & 0 \end{array} \right|$$

$$(x-1)(x^2+x-6) = (x-1)(x+3)(x-2) \quad \left| \quad (x+1)(x^2-x-2) = (x+1)^2(x-2) \right.$$

$$\text{b) } 2 \left| \begin{array}{ccc|c} 1 & -5 & -4 & 20 \\ & 2 & -6 & -20 \\ 1 & -3 & -10 & 0 \end{array} \right|$$

$$3 \left| \begin{array}{ccc|c} 1 & -7 & 0 & 36 \\ & 3 & -12 & -36 \\ 1 & -4 & -12 & 0 \end{array} \right|$$

$$(x-2)(x^2-3x-10) = (x-2)(x-5)(x+2) \quad \left| \quad (x-3)(x^2-4x-12) = (x-3)(x-6)(x+2) \right.$$

Exercice 2.

$$\text{a) } -1 \left| \begin{array}{ccc|c} 1 & -4 & 1 & 6 \\ & -1 & 5 & -6 \\ 1 & -5 & 6 & 0 \end{array} \right|$$

$$1 \left| \begin{array}{ccc|c} 1 & 5 & 2 & -8 \\ & 1 & 6 & 8 \\ 1 & 6 & 8 & 0 \end{array} \right|$$

$$(x+1)(x^2-5x+6) = 0$$

$$(x-1)(x^2+6x+8) = 0$$

$$(x+1)(x-2)(x-3) = 0 \Rightarrow \mathcal{S} = \{-1; 2; 3\}$$

$$(x-1)(x+2)(x+4) = 0 \Rightarrow \mathcal{S} = \{-4; -2; 1\}$$

$$\text{b) } x^3 - 5x^2 + 4x = 0 \Leftrightarrow x(x^2 - 5x + 4) = 0$$

$$x^3 - 3x^2 + 2x = 0 \Leftrightarrow x(x^2 - 3x + 2) = 0$$

$$\Leftrightarrow x(x-4)(x-1) = 0$$

$$\Leftrightarrow x(x-2)(x-1) = 0$$

$$\Rightarrow \mathcal{S} = \{0; 1; 4\}$$

$$\Rightarrow \mathcal{S} = \{0; 1; 2\}$$

Exercice 3.

$$4 \left| \begin{array}{cccc|c} 1 & -2 & -13 & 14 & 24 \\ & 4 & 8 & -20 & -24 \\ 1 & 2 & -5 & -6 & 0 \end{array} \right.$$

$$p(x) = (x - 4)(x^3 + 2x^2 - 5x - 6)$$

$$-1 \left| \begin{array}{cccc|c} 1 & 2 & -5 & -6 \\ & -1 & -1 & 6 \\ 1 & 1 & -6 & 0 \end{array} \right.$$

$$p(x) = (x - 4)(x + 1)(x^2 + x - 6)$$

$$p(x) = (x - 4)(x + 1)(x + 3)(x - 2)$$

$$5 \left| \begin{array}{cccc|c} 1 & 1 & -25 & -37 & 60 \\ & 5 & 30 & 25 & -60 \\ 1 & 6 & 5 & -12 & 0 \end{array} \right.$$

$$p(x) = (x - 5)(x^3 + 6x^2 + 5x - 12)$$

$$1 \left| \begin{array}{ccc|c} 1 & 6 & 5 & -12 \\ & 1 & 7 & 12 \\ 1 & 7 & 12 & 0 \end{array} \right.$$

$$p(x) = (x - 5)(x - 1)(x^2 + 7x + 12)$$

$$p(x) = (x - 5)(x - 1)(x + 3)(x + 4)$$

Exercice 4.

$$\text{a) } r = p(1) = 10 + 7 - 8 = 9$$

$$\text{b) } p(3) = 36 - 9 + a = 0 \Leftrightarrow a = -27$$

$$r = p(1) = 13 + 8 - 4 = 17$$

$$p(2) = 12 - 10 + a = 0 \Leftrightarrow a = -2$$