

Le cercle

3.3.1

a) $C(5; -2) \quad r = 5$

c) $C(5; -2) \quad r = 0$ (cercle-point)

b) $C(-2; 0) \quad r = 8$

d) $C(0; 5) \quad r = \sqrt{5}$

e) $x^2 - 2x + 1 + y^2 + 4y + 4 = 20 + 1 + 4 \Rightarrow (x - 1)^2 + (y + 2)^2 = 25$

$C(1; -2) \quad r = 5$

f) $x^2 - 2x + 1 + y^2 + 4y + 4 = -14 + 1 + 4 \Rightarrow (x - 1)^2 + (y + 2)^2 = -9$

Ce n'est pas un cercle !

g) $x^2 + 4x + 4 + y^2 - 2y + 1 = -5 + 1 + 4 \Rightarrow (x + 2)^2 + (y - 1)^2 = 0$

$C(-2; 1) \quad r = 0$ (cercle-point)

h) $x^2 + x + \frac{1}{4} + y^2 = 0 + \frac{1}{4} \Rightarrow \left(x + \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$

$C\left(-\frac{1}{2}; 0\right) \quad r = \frac{1}{2}$

i) $x^2 + 6x + 9 + y^2 - 4y + 4 = -14 + 9 + 4 \Rightarrow (x + 3)^2 + (y - 2)^2 = -1$

Ce n'est pas un cercle !

j) $x^2 + y^2 + y + \frac{1}{4} = 0 + \frac{1}{4} \Rightarrow x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$

$C\left(0; -\frac{1}{2}\right) \quad r = \frac{1}{2}$

k) $x^2 - \frac{3}{2}x + \frac{9}{16} + y^2 + y + \frac{1}{4} = -\frac{17}{80} + \frac{9}{16} + \frac{1}{4} \Rightarrow \left(x - \frac{3}{4}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{3}{5}$

$C\left(\frac{3}{4}; -\frac{1}{2}\right) \quad r = \frac{\sqrt{15}}{5} = \sqrt{0.6}$

l) $x^2 - \frac{3}{2}x + \frac{9}{16} + y^2 + \frac{4}{3}y + \frac{4}{9} = -\frac{145}{144} + \frac{9}{16} + \frac{4}{9} \Rightarrow \left(x - \frac{3}{4}\right)^2 + \left(y + \frac{2}{3}\right)^2 = 0$

$$C\left(\frac{3}{4}; -\frac{2}{3}\right) \quad r = 0 \text{ (cercle-point)}$$

3.3.2

a) $x^2 + y^2 = 9$

b) $(x-2)^2 + (y+3)^2 = 49$

c) $\|\overrightarrow{OC}\| = \sqrt{6^2 + (-8)^2} = 10 \Rightarrow (x-6)^2 + (y+8)^2 = 100$

d) $(\gamma) : (x+1)^2 + (y-2)^2 = r^2 \quad A \in \gamma \Rightarrow 9 + 16 = 25 = r^2$
 $\Rightarrow (x+1)^2 + (y-2)^2 = 25$

e) M milieu de $AB : M\left(\frac{3-1}{2}; \frac{2+6}{2}\right) \Rightarrow M(1; 4)$

$(\gamma) : (x-1)^2 + (y-4)^2 = r^2 \quad A \in \gamma \Rightarrow 4 + 4 = 8 = r^2$
 $\Rightarrow (x-1)^2 + (y-4)^2 = 8$

f) $\delta(O; d) = \frac{|0-0+20|}{\sqrt{3^2 + (-4)^2}} = \frac{20}{5} = 4 = r \Rightarrow x^2 + y^2 = 16$

g) $\delta(C; d) = \frac{|5+12+9|}{\sqrt{5^2 + (-12)^2}} = \frac{26}{13} = 2 = r \Rightarrow (x-1)^2 + (y+1)^2 = 4$

h) $y_C = 3x_C - 2 \Rightarrow (\gamma) : (x-x_C)^2 + (y-3x_C+2)^2 = r^2$

$A \in \gamma \Rightarrow (3-x_C)^2 + (3-3x_C)^2 = r^2 \Rightarrow 9-6x_C+x_C^2+9-18x_C+9x_C^2 = r^2$

$B \in \gamma \Rightarrow (-1-x_C)^2 + (5-3x_C)^2 = r^2 \Rightarrow 1+2x_C+x_C^2+25-30x_C+9x_C^2 = r^2$

$$\begin{cases} 10x_C^2 - 24x_C + 18 = r^2 \\ 10x_C^2 - 28x_C + 26 = r^2 \end{cases} \Rightarrow \frac{-10x_C^2 + 24x_C - 18 = -r^2}{10x_C^2 - 28x_C + 26 = r^2} \Rightarrow \frac{-10x_C^2 + 24x_C - 18 = -r^2}{-4x_C + 8 = 0}$$

$\Rightarrow 4x_C = 8 \Rightarrow x_C = 2 \Rightarrow r^2 = 40 - 48 + 18 = 10 \Rightarrow y_C = 6 - 2 = 4$

$\Rightarrow (x-2)^2 + (y-4)^2 = 10$

i) M milieu de $AB : M\left(\frac{1+1}{2}; \frac{1-1}{2}\right) \Rightarrow M(1; 0)$

N milieu de $AC : M\left(\frac{1+2}{2}; \frac{1-0}{2}\right) \Rightarrow N\left(\frac{3}{2}; \frac{1}{2}\right)$

$\overrightarrow{AB} = \begin{pmatrix} 1-1 \\ -1-1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \quad \overrightarrow{AC} = \begin{pmatrix} 2-1 \\ 0-1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$(m_{AB}) : y + c_1 = 0 \quad M \in m_{AB} \Rightarrow 0 + c_1 = 0 \Rightarrow c_1 = 0$

$\Rightarrow (m_{AB}) : y = 0$

$(m_{AC}) : x - y + c_2 = 0 \quad N \in m_{AC} \Rightarrow \frac{3}{2} - \frac{1}{2} + c_2 = 0 \Rightarrow c_2 = -1$

$\Rightarrow (m_{AC}) : x - y - 1 = 0$

$m_{AB} \cap m_{AC} : y = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1 \Rightarrow M(1; 0) \text{ (centre du cercle)}$

$(\gamma) : (x-1)^2 + y^2 = r^2 \quad A \in \gamma \Rightarrow 0 + 1 = r^2 \Rightarrow r^2 = 1$

$\Rightarrow (\gamma) : (x-1)^2 + y^2 = 1$

3.3.3

$$\text{a) } x^2 - 3x + \frac{9}{4} + y^2 + 2y + 1 = 3 + \frac{9}{4} + 1 \Rightarrow \left(x - \frac{3}{2}\right)^2 + (y + 1)^2 = \frac{25}{4}$$

$$\Rightarrow C\left(\frac{3}{2}; -1\right) \quad r = \frac{5}{2} \text{ u}$$

$$\delta(C; d) = \frac{|3 + 1 - 3|}{\sqrt{2^2 + (-1)^2}} = \frac{1}{\sqrt{5}} < \frac{5}{2} \Rightarrow \text{la droite coupe le cercle.}$$

$$\text{b) } x^2 - 8x + 16 + y^2 + 2y + 1 = -12 + 16 + 1 \Rightarrow (x - 4)^2 + (y + 1)^2 = 5$$

$$\Rightarrow C(4; -1) \quad r = \sqrt{5} \text{ u}$$

$$\delta(C; d) = \frac{|4 + 2 - 1|}{\sqrt{1^2 + (-2)^2}} = \frac{5}{\sqrt{5}} = \sqrt{5} = r \Rightarrow \text{la droite est tangente au cercle.}$$

$$\text{c) } C(0; 0) \quad r = 1 \text{ u}$$

$$\delta(C; d) = \frac{|0 - 0 + 10|}{\sqrt{1^2 + (-1)^2}} = \frac{10}{\sqrt{2}} > 1 \Rightarrow \text{la droite et le cercle sont disjoints.}$$

3.3.4

$$x^2 - 16x + 64 + y^2 - 20y + 100 = -115 + 64 + 100 \Rightarrow (\gamma_1) : (x - 8)^2 + (y - 10)^2 = 49$$

$$\Rightarrow C_1(8; 10) \quad r_1 = 7 \text{ u}$$

$$x^2 + 8x + 16 + y^2 - 10y + 25 = -5 + 16 + 25 \Rightarrow (\gamma_2) : (x + 4)^2 + (y - 5)^2 = 36$$

$$\Rightarrow C_2(-4; 5) \quad r_2 = 6 \text{ u}$$

$$\overrightarrow{C_1C_2} = \begin{pmatrix} -4 - 8 \\ 5 - 10 \end{pmatrix} = \begin{pmatrix} -12 \\ -5 \end{pmatrix} \Rightarrow \|\overrightarrow{C_1C_2}\| = \sqrt{(-12)^2 + (-5)^2} = 13 = r_1 + r_2$$

$\Rightarrow$ les cercles sont tangents extérieurement.