

Asymptotes

2.7.1

$$\text{a) } f(x) = \frac{x^2 + x - 2}{x^2 + 2x - 3} = \frac{(x+2)(x-1)}{(x+3)(x-1)} \Rightarrow ED(f) = \mathbb{R} \setminus \{-3; 1\}$$

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 2}{x^2 + 2x - 3} \stackrel{\frac{4}{0}}{=} \begin{cases} < +\infty \\ > -\infty \end{cases} \Rightarrow AV : x = -3$$

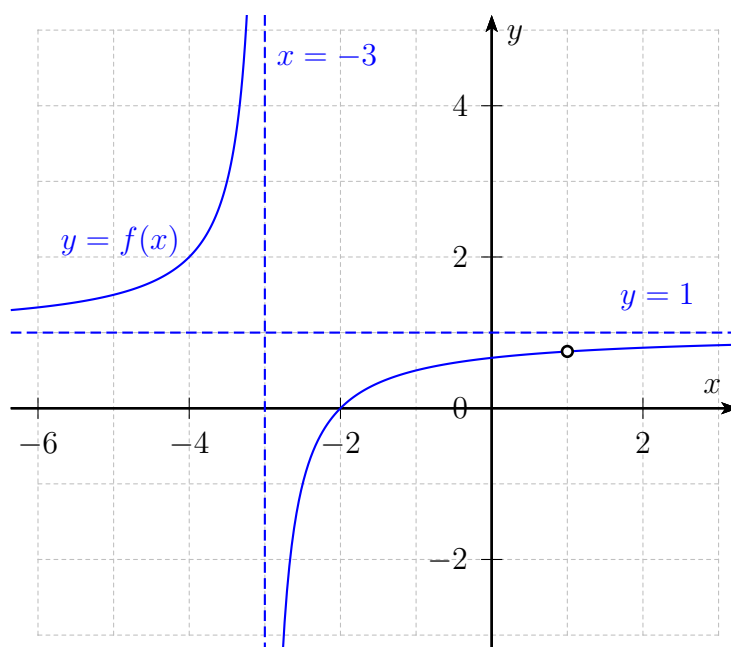
$$\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 + 2x - 3} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(x+3)(x-1)} = \lim_{x \rightarrow 1} \frac{x+2}{x+3} = \frac{3}{4}$$

$$\Rightarrow \text{trou} : \left(1; \frac{3}{4}\right)$$

$$\lim_{x \rightarrow \infty} \frac{x^2 + x - 2}{x^2 + 2x - 3} = \lim_{x \rightarrow \infty} \frac{x^2}{x^2} = 1 \Rightarrow AH : y = 1$$

$$\delta(x) = \frac{x^2 + x - 2}{x^2 + 2x - 3} - 1 = \frac{-x + 1}{x^2 + 2x - 3}$$

x	$-\infty$	-3	1	$+\infty$
$\delta(x)$	+	-	-	
position	dessus	dessous	dessous	



$$\text{b) } f(x) = \frac{x^2 + x + 1}{x - 2} \Rightarrow ED(f) = \mathbb{R} \setminus \{2\}$$

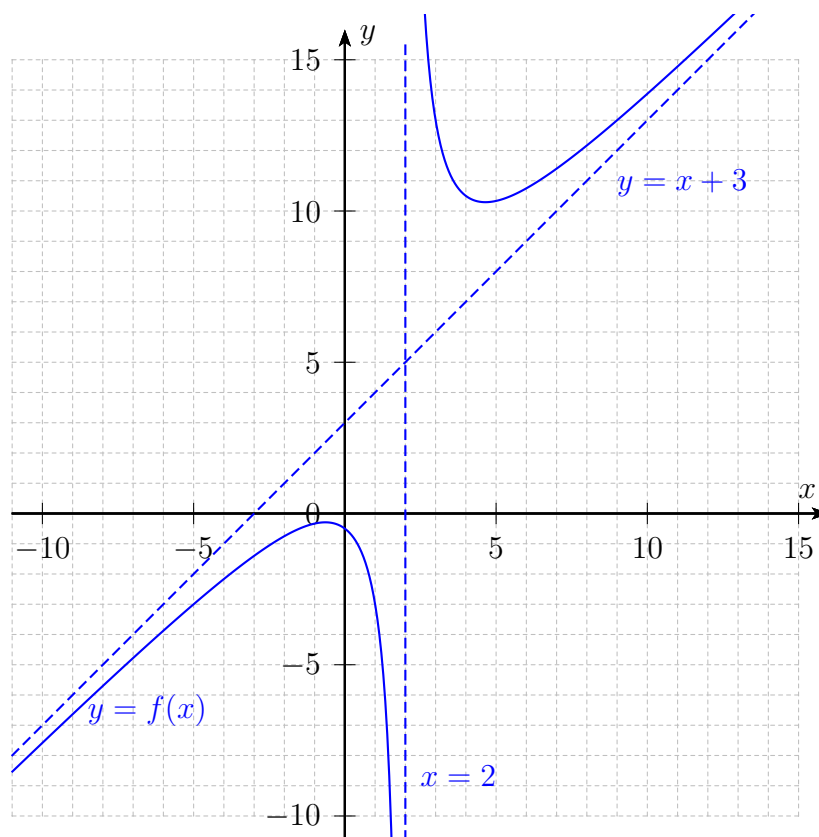
$$\lim_{x \rightarrow 2} \frac{x^2 + x + 1}{x - 2} \overset{\frac{7}{0}}{=} \begin{cases} < -\infty \\ > +\infty \end{cases} \Rightarrow AV : x = 2$$

$$\lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x^2}{x^2} = 1$$

$$\lim_{x \rightarrow \infty} (f(x) - x) = \lim_{x \rightarrow \infty} \frac{3x + 1}{x - 2} = \lim_{x \rightarrow \infty} \frac{3x}{x} = 3 \Rightarrow AO : y = x + 3$$

$$\delta(x) = \frac{x^2 + x + 1}{x - 2} - (x + 3) = \frac{7}{x - 2}$$

x	$-\infty$	2	$+\infty$
$\delta(x)$	$-$		$+$
position	dessous		dessus



$$c) f(x) = \frac{2x^2 + 6x}{(x+2)^2} \Rightarrow ED(f) = \mathbb{R} \setminus \{-2\}$$

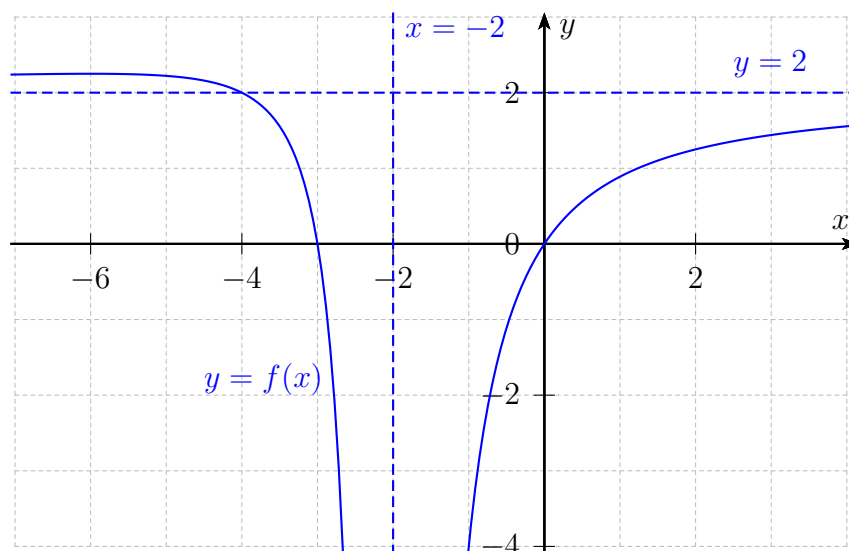
$$\lim_{x \rightarrow -2} \frac{2x^2 + 6x}{(x+2)^2} \stackrel{\frac{-4}{0+}}{=} -\infty \Rightarrow AV : x = -2$$

$$\lim_{x \rightarrow \infty} \frac{2x^2 + 6x}{(x+2)^2} = \lim_{x \rightarrow \infty} \frac{2x^2}{x^2} = 2 \Rightarrow AH : y = 2$$

$$\delta(x) = \frac{2x^2 + 6x}{(x+2)^2} - 2 = \frac{-2x - 8}{(x+2)^2}$$

x	$-\infty$	-4	-2	$+\infty$
$\delta(x)$	+	0	-	-
position	dessus $\cap$ dessous			dessous

intersection avec l'AH : $(-4; 2)$



$$d) f(x) = \frac{x^3}{4 - x^2} = \frac{x^3}{(2-x)(2+x)} \Rightarrow ED(f) = \mathbb{R} \setminus \{-2; 2\}$$

$$\lim_{x \rightarrow -2} \frac{x^3}{4 - x^2} \stackrel{\frac{-8}{0}}{=} \begin{cases} < +\infty \\ > -\infty \end{cases} \Rightarrow AV : x = -2$$

$$\lim_{x \rightarrow 2} \frac{x^3}{4 - x^2} \stackrel{\frac{8}{0}}{=} \begin{cases} < +\infty \\ > -\infty \end{cases} \Rightarrow AV : x = 2$$

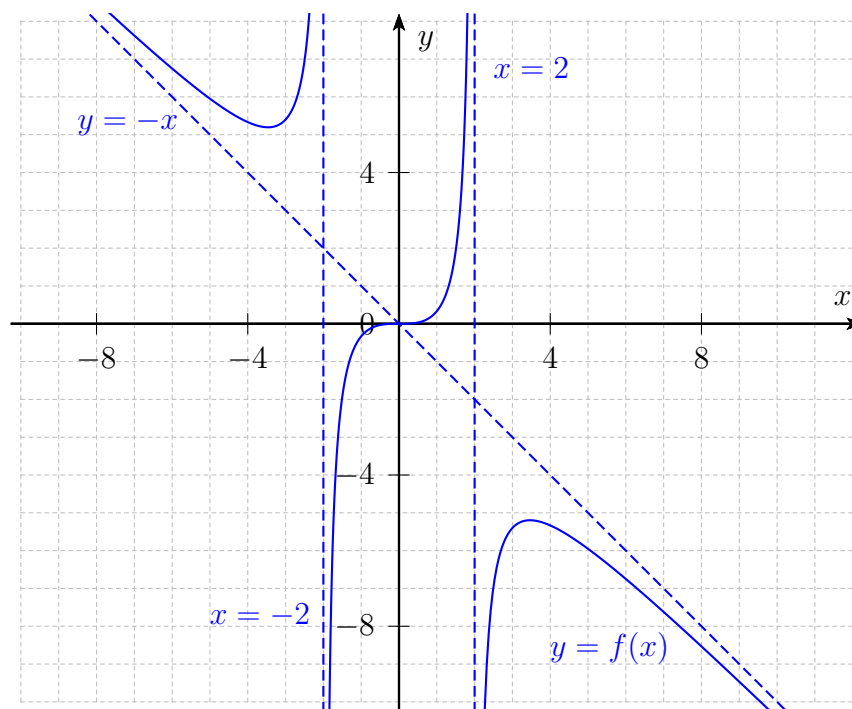
$$\lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x^3}{-x^3} = -1$$

$$\lim_{x \rightarrow \infty} (f(x) + x) = \lim_{x \rightarrow \infty} \frac{4x}{4 - x^2} = \lim_{x \rightarrow \infty} \frac{4x}{-x^2} = 0 \Rightarrow \text{AO} : y = -x$$

$$\delta(x) = \frac{x^3}{4 - x^2} + x = \frac{4x}{4 - x^2}$$

x	$-\infty$	-2	0	-2	$+\infty$
$\delta(x)$	+	-	0	+	-
position	dessus	dessous	$\cap$	dessus	dessous

inter. avec l'AH : (0;0)



$$\text{e) } f(x) = \frac{x^2 - 4x}{x^2 - 4x + 3} = \frac{x(x - 4)}{(x - 3)(x - 1)} \Rightarrow \text{ED}(f) = \mathbb{R} \setminus \{1; 3\}$$

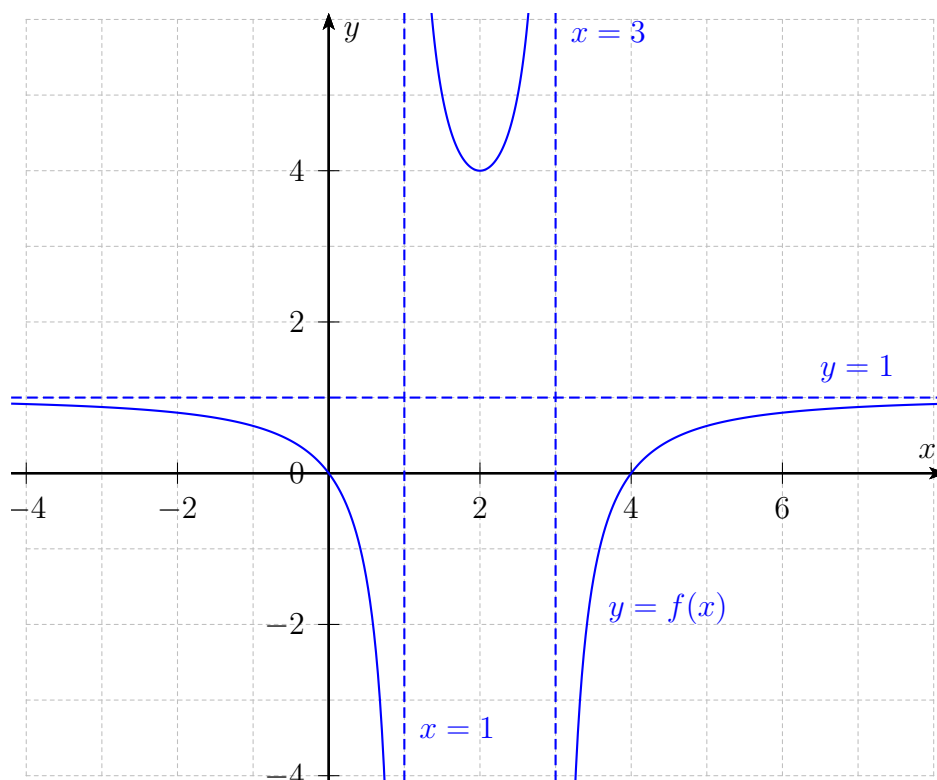
$$\lim_{x \rightarrow 1} \frac{x^2 - 4x}{x^2 - 4x + 3} \underset{\substack{= \\ \frac{-3}{0}}}{\begin{matrix} < \\ > \end{matrix}} \begin{matrix} -\infty \\ +\infty \end{matrix} \Rightarrow \text{AV} : x = 1$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x}{x^2 - 4x + 3} \underset{\substack{= \\ \frac{-3}{0}}}{\begin{matrix} < \\ > \end{matrix}} \begin{matrix} +\infty \\ -\infty \end{matrix} \Rightarrow \text{AV} : x = 3$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - 4x}{x^2 - 4x + 3} = \lim_{x \rightarrow \infty} \frac{x^2}{x^2} = 1 \Rightarrow \text{AH} : y = 1$$

$$\delta(x) = \frac{x^2 - 4x}{x^2 - 4x + 3} - 1 = \frac{-3}{x^2 - 4x + 3}$$

x	$-\infty$	1	3	$+\infty$
$\delta(x)$	-	+	-	
position	dessous	dessus	dessous	



$$f) \quad f(x) = \frac{x^3 - 3x^2 + 2}{x^2 - 4} = \frac{x^3 - 3x^2 + 2}{(x-2)(x+2)} \quad \Rightarrow \quad ED(f) = \mathbb{R} \setminus \{-2; 2\}$$

$$\lim_{x \rightarrow -2} \frac{x^3 - 3x^2 + 2}{x^2 - 4} \underset{\frac{-18}{0}}{=} \begin{cases} < -\infty \\ > +\infty \end{cases} \quad \Rightarrow \quad AV : x = -2$$

$$\lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 2}{x^2 - 4} \underset{\frac{-2}{0}}{=} \begin{cases} < +\infty \\ > -\infty \end{cases} \quad \Rightarrow \quad AV : x = 2$$

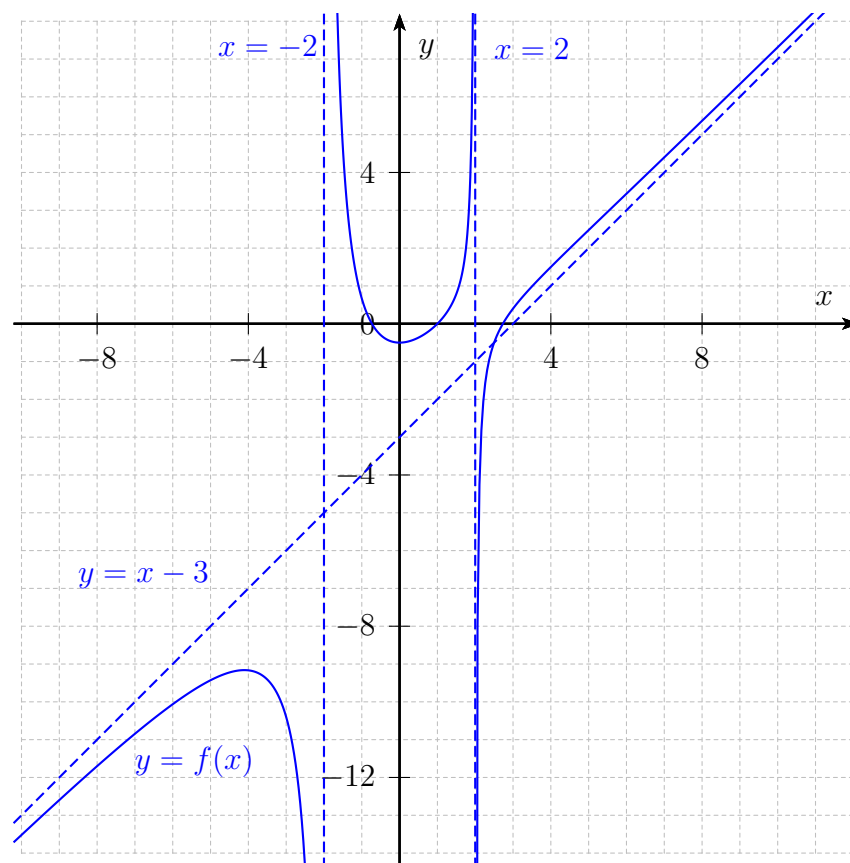
$$\lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x^3}{x^3} = 1$$

$$\lim_{x \rightarrow \infty} (f(x) - x) = \lim_{x \rightarrow \infty} \frac{-3x^2 + 6}{x^2 - 4} = \lim_{x \rightarrow \infty} \frac{-3x^2}{x^2} = -3 \quad \Rightarrow \quad AO : y = x - 3$$

$$\delta(x) = \frac{x^3 - 3x^2 + 2}{x^2 - 4} - (x - 3) = \frac{4x - 10}{x^2 - 4}$$

x	$-\infty$	-2	2	$\frac{5}{2}$	$+\infty$
$\delta(x)$	$-$	$+$	$-$	0	$+$
position	dessous	dessus	dessous $\cap$ dessus		

inter. avec l'AH : $\left(\frac{5}{2}; -\frac{1}{2}\right)$



2.7.2

$$\text{a) } f(x) = x\sqrt{\frac{x}{x+1}} \quad \sqrt{\frac{x}{x+1}} \geq 0 \Rightarrow ED(f) =]-\infty; -1[\cup [0; +\infty[$$

$$\lim_{x \rightarrow -1^-} x\sqrt{\frac{x}{x+1}} \stackrel{\frac{-1}{0^+}}{=} -\infty \Rightarrow \text{AV : } x = -1$$

$$\lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \sqrt{\frac{x}{x+1}} = \lim_{x \rightarrow \infty} \sqrt{\frac{x}{x}} = 1$$

$$\lim_{x \rightarrow \infty} (f(x) - x) = \lim_{x \rightarrow \infty} x \left(\sqrt{\frac{x}{x+1}} - 1 \right) = \lim_{x \rightarrow \infty} x \left(\sqrt{\frac{x}{x+1}} - 1 \right) \cdot \frac{\sqrt{\frac{x}{x+1}} + 1}{\sqrt{\frac{x}{x+1}} + 1}$$

$$= \lim_{x \rightarrow \infty} x \cdot \frac{\frac{x}{x+1} - 1}{\sqrt{\frac{x}{x+1}} + 1} = \lim_{x \rightarrow \infty} \frac{\frac{-x}{x+1}}{\sqrt{\frac{x}{x+1}} + 1} = \lim_{x \rightarrow \infty} \frac{\frac{-x}{x}}{\sqrt{\frac{x}{x}} + 1} = -\frac{1}{2} \Rightarrow \text{AO : } y = x - \frac{1}{2}$$