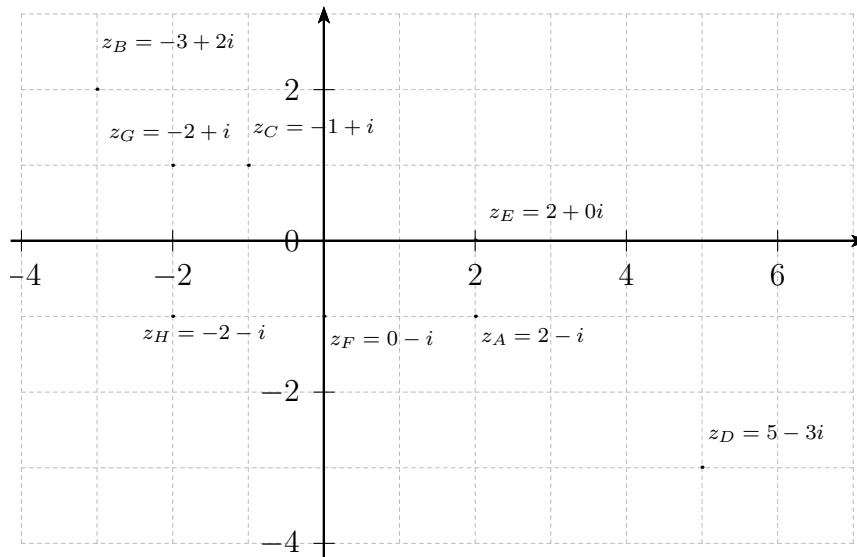


Nombres complexes sous forme trigonométrique

1.2.1



1.2.2

$$\text{a) } |z| = 1 \quad \cos(\theta) = 1 \quad \sin(\theta) = 0 \quad \Rightarrow \quad z = 1 \operatorname{cis} 0$$

$$\text{b) } |z| = 1 \quad \cos(\theta) = 0 \quad \sin(\theta) = 1 \quad \Rightarrow \quad z = 1 \operatorname{cis} \frac{\pi}{2}$$

$$\text{c) } |z| = 2 \quad \cos(\theta) = -1 \quad \sin(\theta) = 0 \quad \Rightarrow \quad z = 2 \operatorname{cis} \pi$$

$$\text{d) } |z| = \sqrt{2} \quad \cos(\theta) = -\frac{\sqrt{2}}{2} \quad \sin(\theta) = -\frac{\sqrt{2}}{2} \quad \Rightarrow \quad z = \sqrt{2} \operatorname{cis} \frac{5\pi}{4}$$

$$\text{e) } |z| = 2 \quad \cos(\theta) = -\frac{1}{2} \quad \sin(\theta) = -\frac{\sqrt{3}}{2} \quad \Rightarrow \quad z = 2 \operatorname{cis} \frac{4\pi}{3}$$

$$\text{f) } |z| = 5 \quad \cos(\theta) = \frac{3}{5} \quad \sin(\theta) = \frac{4}{5} \quad \Rightarrow \quad z = 5 \operatorname{cis} 53.13^\circ$$

1.2.3

$$\text{a) } z = 4 \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) = 2 - 2\sqrt{3}i$$

$$\text{b) } z = \frac{3}{4} \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) = \frac{-3\sqrt{2}}{8} + \frac{3\sqrt{2}i}{8}$$

$$\text{c) } z = \pi(-1 + 0i) = -\pi$$

$$\text{d) } z = 4 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = 2 + 2\sqrt{3}i$$

$$\text{e) } z = 1(0 - i) = -i$$

$$\text{f) } z = 1(-1 + 0i) = -1$$

1.2.4

$$\text{a) } (2 \cdot 3) \operatorname{cis} \left(\frac{\pi}{4} + \frac{\pi}{6} \right) \Rightarrow \left[6; \frac{5\pi}{12} \right]$$

$$\text{b) } (6 : 3) \operatorname{cis} \left(\frac{2\pi}{3} + \frac{\pi}{3} \right) \Rightarrow [2; \pi]$$

$$\text{c) } (2^3) \operatorname{cis} \left(3 \cdot \frac{\pi}{3} \right) \Rightarrow [8; \pi]$$

1.2.5

$$\text{a) } |z_1| = \sqrt{2} \quad \cos(\theta_1) = -\frac{\sqrt{2}}{2} \quad \sin(\theta_1) = \frac{\sqrt{2}}{2} \Rightarrow z_1 = \sqrt{2} \operatorname{cis} \frac{3\pi}{4}$$

$$|z_2| = \sqrt{2} \quad \cos(\theta_2) = \frac{\sqrt{2}}{2} \quad \sin(\theta_2) = \frac{\sqrt{2}}{2} \Rightarrow z_2 = \sqrt{2} \operatorname{cis} \frac{\pi}{4}$$

$$\Rightarrow z_1 z_2 = 2 \operatorname{cis} \pi = -2, \quad \frac{z_1}{z_2} = 1 \operatorname{cis} \frac{\pi}{2} = i$$

$$\text{b) } |z_1| = 4 \quad \cos(\theta_1) = -\frac{1}{2} \quad \sin(\theta_1) = -\frac{\sqrt{3}}{2} \Rightarrow z_1 = 4 \operatorname{cis} \frac{4\pi}{3}$$

$$|z_2| = 5 \quad \cos(\theta_2) = 0 \quad \sin(\theta_2) = 1 \Rightarrow z_2 = 5 \operatorname{cis} \frac{\pi}{2}$$

$$\Rightarrow z_1 z_2 = 20 \operatorname{cis} \frac{11\pi}{6} = 10\sqrt{3} - 10i, \quad \frac{z_1}{z_2} = \frac{4}{5} \operatorname{cis} \frac{5\pi}{6} = -\frac{2\sqrt{3}}{5} + \frac{2}{5}i$$

$$\text{c) } |z_1| = 2 \quad \cos(\theta_1) = 0 \quad \sin(\theta_1) = 1 \Rightarrow z_1 = 2 \operatorname{cis} \frac{\pi}{2}$$

$$|z_2| = 3 \quad \cos(\theta_2) = 0 \quad \sin(\theta_2) = -1 \Rightarrow z_2 = 3 \operatorname{cis} \frac{3\pi}{2}$$

$$\Rightarrow z_1 z_2 = 6 \operatorname{cis} 0 = 6, \quad \frac{z_1}{z_2} = \frac{2}{3} \operatorname{cis} \pi = -\frac{2}{3}$$

$$\text{d) } z_1 z_2 = 40, \quad \frac{z_1}{z_2} = \frac{5}{2}$$

1.2.6

$$\text{a) } \sqrt{13}$$

$$\text{d) } 3$$

$$\text{g) } 6$$

$$\text{b) } \sqrt{2}$$

$$\text{e) } \frac{1}{4} + \frac{3}{4} = 1$$

$$\text{h) } 0$$

$$\text{c) } 2$$

$$\text{f) } 5$$

$$\text{i) } \cos^2(t) + \sin^2(t) = 1$$

1.2.7

$$z = 1 \operatorname{cis} \theta = \cos(\theta) + i \sin(\theta)$$

$$\Rightarrow z^2 = \underbrace{1 \operatorname{cis} 2\theta}_{\cos(2\theta) + i \sin(2\theta)} = \cos^2(\theta) - \sin^2(\theta) + i 2 \sin(\theta) \cos(\theta)$$

$$\Rightarrow \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) \quad \sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\begin{aligned} \Rightarrow \cos(4\theta) &= \cos(2 \cdot 2\theta) = \cos^2(2\theta) - \sin^2(2\theta) = (\cos^2(\theta) - \sin^2(\theta))^2 - (2 \sin(\theta) \cos(\theta))^2 \\ &= \cos^4(\theta) - 2 \sin^2(\theta) \cos^2(\theta) + \sin^4(\theta) - 4 \sin^2(\theta) \cos^2(\theta) = \cos^4(\theta) - 6 \sin^2(\theta) \cos^2(\theta) + \sin^4(\theta) \end{aligned}$$

$$\begin{aligned} \Rightarrow \sin(4\theta) &= \sin(2 \cdot 2\theta) = 2 \sin(2\theta) \cos(2\theta) = 2 \cdot 2 \sin(\theta) \cos(\theta) \cdot (\cos^2(\theta) - \sin^2(\theta)) \\ &= 4 \cos^2(\theta) \sin(\theta) - 4 \cos(\theta) \sin^3(\theta) \end{aligned}$$

1.2.8

$$\text{a) } z^4 = r^4 \operatorname{cis} 4\theta = \sqrt{2} \operatorname{cis} \frac{\pi}{4} \Rightarrow r^4 = \sqrt{2} \quad 4\theta = \frac{\pi}{4} + k \cdot 2\pi$$

$$\Rightarrow z_1 = \sqrt[8]{2} \operatorname{cis} \frac{\pi}{16}, z_2 = \sqrt[8]{2} \operatorname{cis} \frac{9\pi}{16}, z_3 = \sqrt[8]{2} \operatorname{cis} \frac{17\pi}{16}, z_4 = \sqrt[8]{2} \operatorname{cis} \frac{25\pi}{16}$$

$$\text{b) } z^2 = a + bi \Rightarrow \begin{cases} a^2 + b^2 = 25 \\ a^2 - b^2 = -7 \\ ab = 12 \end{cases} \Rightarrow 2a^2 = 18 \Rightarrow a^2 = 9$$

$$\Rightarrow a = \pm 3 \Rightarrow b = \pm 4 \Rightarrow z^2 = \pm 3 \pm 4i$$

$$z = c + di \Rightarrow \begin{cases} c^2 + d^2 = 5 \\ c^2 - d^2 = 3 \\ cd = 2 \end{cases} \Rightarrow 2c^2 = 8 \Rightarrow c^2 = 4$$

$$\Rightarrow c = \pm 2 \Rightarrow d = \pm 1 \Rightarrow z_1 = 2 + i \quad z_2 = -2 - i$$

$$z = e + fi \Rightarrow \begin{cases} e^2 + f^2 = 5 \\ e^2 - f^2 = -3 \\ ef = -2 \end{cases} \Rightarrow 2e^2 = 2 \Rightarrow e^2 = 1$$

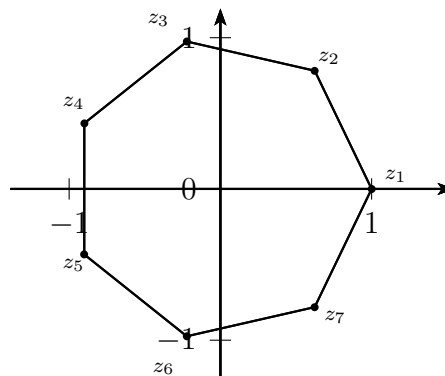
$$\Rightarrow e = \pm 1 \Rightarrow f = \mp 2 \Rightarrow z_3 = 1 - 2i \quad z_4 = -1 + 2i$$

1.2.9

$$\text{a) } z^7 = 1 \operatorname{cis} 0 \Rightarrow r^7 = 1 \quad 7\theta = k \cdot 2\pi$$

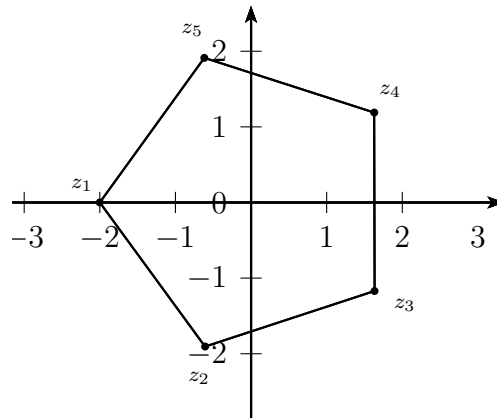
$$z_1 = 1 \operatorname{cis} 0, z_2 = 1 \operatorname{cis} \frac{2\pi}{7}, z_3 = 1 \operatorname{cis} \frac{4\pi}{7}, z_4 = 1 \operatorname{cis} \frac{6\pi}{7}, z_5 = 1 \operatorname{cis} \frac{8\pi}{7},$$

$$z_6 = 1 \operatorname{cis} \frac{10\pi}{7}, z_7 = 1 \operatorname{cis} \frac{12\pi}{7}$$



$$\text{b) } z^5 = 32 \operatorname{cis} \pi \Rightarrow r^5 = 32 \quad 5\theta = \pi + k \cdot 2\pi$$

$$z_1 = 2 \operatorname{cis} \frac{\pi}{5}, \quad z_2 = 2 \operatorname{cis} \frac{3\pi}{5}, \quad z_3 = 2 \operatorname{cis} \pi, \quad z_4 = 2 \operatorname{cis} \frac{7\pi}{5}, \quad z_5 = 2 \operatorname{cis} \frac{9\pi}{5}$$



1.2.10

a) ± 1

$$\text{b) } \begin{cases} a^2 + b^2 = 1 \\ a^2 - b^2 = 0 \\ ab = \frac{1}{2} \end{cases} \Rightarrow 2a^2 = 1 \Rightarrow a^2 = \frac{1}{2}$$

$$\Rightarrow a = \pm \frac{\sqrt{2}}{2} \Rightarrow b = \pm \frac{\sqrt{2}}{2} \Rightarrow z = \pm \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right)$$

$$\text{c) } \begin{cases} a^2 + b^2 = 1 \\ a^2 - b^2 = 0 \\ ab = -\frac{1}{2} \end{cases} \Rightarrow 2a^2 = 1 \Rightarrow a^2 = \frac{1}{2}$$

$$\Rightarrow a = \pm \frac{\sqrt{2}}{2} \Rightarrow b = \mp \frac{\sqrt{2}}{2} \Rightarrow z = \pm \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right)$$

d) $\pm 3i$

$$\text{e) } \begin{cases} a^2 + b^2 = 5 \\ a^2 - b^2 = 3 \\ ab = 2 \end{cases} \Rightarrow 2a^2 = 8 \Rightarrow a^2 = 4$$

$$\Rightarrow a = \pm 2 \Rightarrow b = \pm 1 \Rightarrow z = \pm (2 + i)$$

$$\text{f) } \begin{cases} a^2 + b^2 = 13 \\ a^2 - b^2 = -5 \\ ab = 6 \end{cases} \Rightarrow 2a^2 = 8 \Rightarrow a^2 = 4$$

$$\Rightarrow a = \pm 2 \Rightarrow b = \pm 3 \Rightarrow z = \pm (2 + 3i)$$