Les dérivées

Exercice 1.

a) $f'(x) = \dfrac{5\cdot(3x-4)^4\cdot 3\cdot(1-x)^4 - (3x-4)^5\cdot 4\cdot(1-x)^3\cdot(-1)}{(1-x)^8}$

$= \dfrac{(3x-4)^4\cdot(1-x)^3[15(1-x)+4(3x-4)]}{(1-x)^8} = \boxed{\dfrac{(3x-4)^4\cdot(-3x-1)}{(1-x)^5}}$

b) $g'(x) = 3\cdot\sqrt{1-4x^2} + 3x\cdot\dfrac{(-8x)}{2\sqrt{1-4x^2}} = 3\cdot\sqrt{1-4x^2} + 3x\cdot\dfrac{(-4x)}{\sqrt{1-4x^2}}$

$= \dfrac{3(1-4x^2)-12x^2}{\sqrt{1-4x^2}} = \boxed{\dfrac{3-24x^2}{\sqrt{1-4x^2}}}$

c) $h'(x) = -2\sin(2x)\cdot\sin^3(x) + \cos(2x)\cdot 3\cdot\sin^2(x)\cdot\cos(x)$

$= \boxed{\sin^2(x)\cdot[-2\sin(2x)\sin(x) + 3\cos(2x)\cos(x)]}$

Exercice 2.

$T(a; a^3+a^2) \qquad (t):\ y = mx + h$

$f'(x) = 3x^2 + 2x \quad\Rightarrow\quad m = f'(a) = 3a^2 + 2a$

$\Rightarrow\quad (t):\ y = (3a^2+2a)x + h$

$A \in t \quad\Rightarrow\quad 0 = \dfrac{3}{5}\cdot(3a^2+2a) + h \quad\Rightarrow\quad h = -\dfrac{9a^2}{5} - \dfrac{6a}{5}$

$\Rightarrow\quad (t):\ y = (3a^2+2a)x - \dfrac{9a^2}{5} - \dfrac{6a}{5}$

$T \in t \quad\Rightarrow\quad a^3 + a^2 = a\cdot(3a^2+2a) - \dfrac{9a^2}{5} - \dfrac{6a}{5} \quad\Rightarrow\quad 2a^3 - \dfrac{4a^2}{5} - \dfrac{6a}{5} = 0$

$\Rightarrow\quad 10a^3 - 4a^2 - 6a = 0 \quad\Rightarrow\quad 2a(5a^2-2a-3) = 0 \quad\Rightarrow\quad 2a(5a+3)(a-1) = 0$

$\Rightarrow\quad a = 0 \qquad a = 1 \qquad a = -\dfrac{3}{5}$

$\Rightarrow\quad \boxed{(t_1):\ y = 0 \qquad (t_2):\ y = 5x - 3 \qquad (t_3):\ y = -\dfrac{3}{25}x + \dfrac{9}{125}}$

Exercice 3.

$T(-1; y_t) \quad\Rightarrow\quad y_t = 1 + 4 = 5 \quad\Rightarrow\quad T(-1;5) \quad\Rightarrow\quad f(-1) = -1 + a - b = 5$

$f'(x) = 3x^2 + 2ax + b \quad\Rightarrow\quad m = -1 = f'(-1) = 3 - 2a + b$

$\Rightarrow\quad \begin{cases} a - b = 6 \\ -2a + b = -4 \end{cases} \quad\Rightarrow\quad -a = 2 \quad\Rightarrow\quad \boxed{a = -2}$

$\Rightarrow\quad \boxed{b = -4 - 4 = -8}$

Exercice 4.

$ED(f) = \mathbb{R} - \{2\}$

x	$-\infty$		-1		2		$+\infty$
$f(x)$		$-$	0	$+$	$\|\|$	$+$	

$\lim\limits_{x \to 2} f(x) = +\infty \quad \Rightarrow \quad \text{AV} : x = 2$

$f(x) = \dfrac{x^3+3x^2+3x+1}{x^2-4x+4} = x + 7 + \dfrac{27x-27}{x^2-4x+4} \quad \Rightarrow \quad \text{AO} : y = x + 7$

$\delta(x) = \dfrac{27x-27}{x^2-4x+4}$

x	$-\infty$		1		2		$+\infty$
$\delta(x)$		$-$	0	$+$	$\|\|$	$+$	
position relative		dessous	$\cap$	dessus	$\|\|$	dessus	

$f'(x) = \dfrac{3(x+1)^2(x-2)^2 - (x+1)^3 \cdot 2(x-2)}{(x-2)^4} = \dfrac{(x-2)(x+1)^2[3(x-2)-2(x+1)]}{(x-2)^4}$

$= \dfrac{(x+1)^2(x-8)}{(x-2)^3} \qquad ED(f') = ED(f)$

x	$-\infty$		-1		2		8		$+\infty$
$f'(x)$		$+$	0	$+$	$\|\|$	$-$	0	$+$	
$f(x)$	$-\infty$	↗		$+\infty$	$\|\|$	$+\infty$ ↘	$\frac{81}{4}$	↗	$+\infty$

minimum : $\left(8; \dfrac{81}{4}\right)$

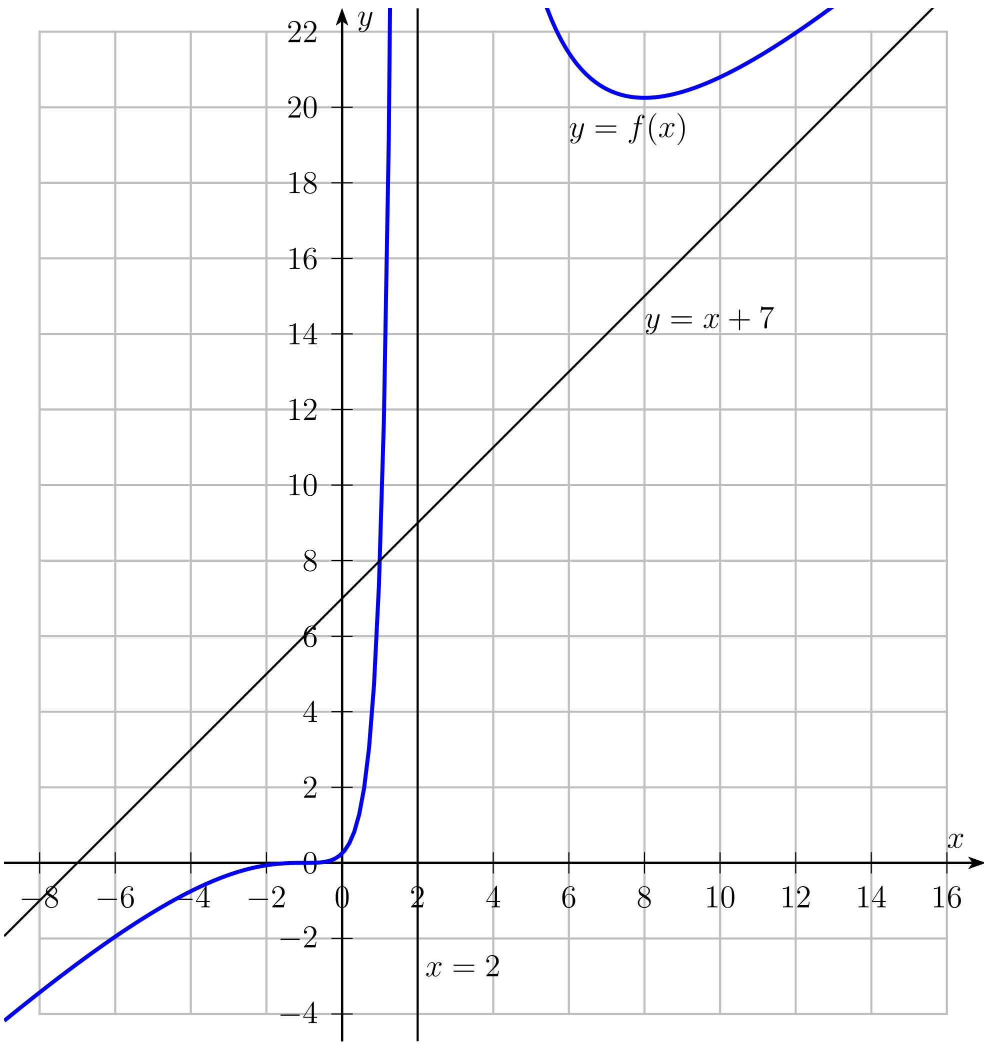
$$f''(x) = \frac{[2(x+1)(x-8)+(x+1)^2](x-2)^3-(x+1)^2(x-8)\cdot 3(x-2)^2}{(x-2)^6}$$

$$= \frac{(x+1)(x-2)^2[(3x-15)(x-2)-3(x+1)(x-8)]}{(x-2)^6} = \frac{(x+1)[3x^2-21x+30-3x^2+21x+24]}{(x-2)^4}$$

$$= \frac{54(x+1)}{(x-2)^4} \qquad ED(f'') = ED(f)$$

x	$-\infty$		-1		2		$+\infty$
$f''(x)$		$-$	0	$+$	‖	$+$	
$f(x)$		$\cap$	0	$\cup$	‖	$\cup$	

point d'inflexion : $(-1;0)$



Exercice 5.

$x + y = 75 \quad \Rightarrow \quad y = 75 - x$

$f(x;y) = (x-10)\cdot y \quad \Rightarrow \quad f(x) = (x-10)(75-x) = -x^2+85x-750 \qquad ED(f) =]0;75[$

$f'(x) = -2x + 85$

x	0		$\frac{85}{2}$		75
$f'(x)$		$+$	0	$-$	
$f(x)$	0	↗	max	↘	0

$y = 75 - \dfrac{85}{2} = \dfrac{65}{2} \quad \Rightarrow \quad y + 10 = \dfrac{85}{2}$

l'aire à l'intérieure du cadre est maximale lorsque les dim. ext. du cadre valent $\dfrac{85}{2}$ cm