Géométrie analytique dans l'espace

Exercice 1.

a) $\overrightarrow{u} \wedge \overrightarrow{v} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \quad \Rightarrow \quad \|\overrightarrow{u} \wedge \overrightarrow{v}\| = 2 \text{ u}$

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \qquad \det(\overrightarrow{AB}; \overrightarrow{u}; \overrightarrow{v}) = -2$

$\Rightarrow \quad \delta(a; b) = \frac{2}{2} = \boxed{1 \text{ u}}$

b) $\boxed{\overrightarrow{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$

c) $(\alpha): \; z + d = 0 \qquad B \in \alpha \quad \Rightarrow \quad 1 + d = 0 \quad \Rightarrow \quad d = -1 \quad \Rightarrow \boxed{(\alpha): \; z - 1 = 0}$

d) $\|\overrightarrow{u}\| = \sqrt{1+9} = \sqrt{10} \qquad \|\overrightarrow{v}\| = \sqrt{4} = 2$

$\cos(\varphi) = \frac{6}{2\sqrt{10}} = \frac{3\sqrt{10}}{10} \quad \Rightarrow \quad \varphi \simeq \boxed{18,43°}$

Exercice 2.

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad \overrightarrow{AC} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \qquad \overrightarrow{n} = \overrightarrow{AB} \wedge \overrightarrow{AC} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$\|\overrightarrow{n}\| = \sqrt{1+4+1} = \sqrt{6} \qquad \|\overrightarrow{d}\| = \sqrt{4+1+1} = \sqrt{6}$

$\cos(\varphi) = \frac{|2-2+1|}{6} = \frac{1}{6} \quad \Rightarrow \quad \varphi \simeq 80,41° \quad \Rightarrow \quad \theta \simeq \boxed{9,59°}$

Exercice 3.

a) $\overrightarrow{n} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad \Rightarrow \quad (n) : \begin{cases} x = 3 + k \\ y = 2 - k \\ z = -1 + 2k \end{cases} \quad \text{avec } k \in \mathbb{R}$

$n \cap \alpha : 3 + k - 2 + k - 2 + 4k = 5 \quad \Leftrightarrow \quad 6k = 6 \quad \Leftrightarrow \quad k = 1 \Rightarrow \boxed{H(4; 1; 1)}$

$\delta(A; \alpha) = \frac{|3-2-2-5|}{\sqrt{1+1+4}} = \frac{6}{\sqrt{6}} = \boxed{\sqrt{6} \text{ u}}$

b) $\overrightarrow{AH} = \begin{pmatrix} 4-3 \\ 1-2 \\ 1+1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad \Rightarrow \quad \overrightarrow{OA'} = \overrightarrow{OA} + 2 \cdot \overrightarrow{AH}$

$$\overrightarrow{OA'} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + 2 \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} \quad \Rightarrow \boxed{A'(5;0;3)}$$