Limites

Exercice 1.

a) $$\lim_{x\to 6} \frac{2-\sqrt{x-2}}{x^2-36} \overbrace{=}^{"\frac{0}{0}" \text{ fi}} \lim_{x\to 6} \frac{4-(x-2)}{(x-6)(x+6)(2+\sqrt{x-2})} = \lim_{x\to 6} \frac{6-x}{(x-6)(x+6)(2+\sqrt{x-2})}$$
$$= \lim_{x\to 6} \frac{-1}{(x+6)(2+\sqrt{x-2})} = \boxed{-\frac{1}{48}}$$

b) $$\lim_{x\to 1} \frac{\sin(x-1)}{x-1} \overbrace{=}^{t=x-1} \lim_{t\to 0} \frac{\sin(t)}{t} = \boxed{1}$$

c) $$\lim_{x\to 1} \frac{x^3-6x^2+12x-7}{x^2+4x-5} \overbrace{=}^{"\frac{0}{0}" \text{ fi}} \lim_{x\to 1} \frac{(x-1)(x^2-5x+7)}{(x+5)(x-1)} = \lim_{x\to 1} \frac{x^2-5x+7}{x+5} = \boxed{\frac{1}{2}}$$

d) $$\lim_{x\to -\infty} \frac{(2-\frac{6}{x})^4}{1-\frac{1}{x}+\frac{1}{x^4}} = \frac{2^4}{1} = \boxed{16}$$

e) $$\lim_{x\to -\infty} \sqrt{x^2-3x-2}-x = \boxed{+\infty}$$

f) $$\lim_{x\to +\infty} \sqrt{x^2-3x-2}-x \overbrace{=}^{"\infty-\infty" \text{ fi}} \lim_{x\to +\infty} \frac{x^2-3x-2-x^2}{\sqrt{x^2-3x-2}+x} = \lim_{x\to +\infty} \frac{-3x-2}{\sqrt{x^2-3x-2}+x}$$
$$= \lim_{x\to +\infty} \frac{-3x}{|x|+x} = \lim_{x\to +\infty} \frac{-3x}{2x} = \boxed{-\frac{3}{2}}$$

Exercice 2.

a) $y = x - 1$

b) $$x-1+\frac{ax+b}{x^2+1} = \frac{(x-1)(x^2+1)+ax+b}{x^2+1} = \frac{x^3-x^2+x-1+ax+b}{x^2+1}$$
$$= \frac{x^3-x^2+(a+1)x+b-1}{x^2+1} = \frac{x(x+1)(x-2)}{x^2+1} = \frac{x^3-x^2-2x}{x^2+1}$$
$$\Rightarrow \quad b-1=0 \quad \Rightarrow \quad b=1$$
$$\Rightarrow \quad a+1=-2 \quad \Rightarrow \quad a=-3$$
$$\Rightarrow \quad f(x) = k\cdot\left(x-1+\frac{-3x+1}{x^2+1}\right) \text{ et } f(1) = k\cdot(0-1) = -k = -1 \quad \Rightarrow \quad k=1$$
$$\Rightarrow \boxed{f(x) = x-1+\frac{-3x+1}{x^2+1}}$$

Exercice 3.

$x^2 - 4x + 3 > 0 \quad \Rightarrow \quad (x-3)(x-1) > 0 \quad \Rightarrow \quad \boxed{ED(f) =]-\infty; 1[\cup]3; +\infty[}$

x	$-\infty$		0		1		3		$+\infty$
$f(x)$		$+$	0	$+$				$+$	

$$\lim_{x \underset{<}{\to} 1} \frac{x^2}{\sqrt{x^2-4x+3}} \overbrace{=}^{\frac{1}{\to 0^+}} +\infty \quad \Rightarrow \quad \boxed{\text{AV} : x = 1}$$

$$\lim_{x \underset{>}{\to} 3} \frac{x^2}{\sqrt{x^2-4x+3}} \overbrace{=}^{\frac{9}{\to 0^+}} +\infty \quad \Rightarrow \quad \boxed{\text{AV} : x = 3}$$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{x^2}{x\sqrt{x^2-4x+3}} = \lim_{x\to-\infty} \frac{x}{|x|} = -1$$

$$\lim_{x\to-\infty} f(x) + x = \lim_{x\to-\infty} \frac{x^2}{\sqrt{x^2-4x+3}} + x = \lim_{x\to-\infty} \frac{x^2 + x\sqrt{x^2-4x+3}}{\sqrt{x^2-4x+3}} =$$

$$\lim_{x\to-\infty} \frac{x^4 - x^2(x^2-4x+3)}{\sqrt{x^2-4x+3}(x^2 - x\sqrt{x^2-4x+3})} = \lim_{x\to-\infty} \frac{x^4 - x^4 + 4x^3 - 3x^2}{\sqrt{x^2-4x+3} \cdot (x^2 - x\sqrt{x^2-4x+3})}$$

$$= \lim_{x\to-\infty} \frac{4x^3}{|x| \cdot (x^2 - x|x|)} = \lim_{x\to-\infty} \frac{4x^3}{(-x) \cdot [x^2 - x(-x)]} = \lim_{x\to-\infty} \frac{4x^3}{(-2x^3)} = -2$$

$$\Rightarrow \quad \boxed{\text{AO} : y = -x - 2 \qquad (x \to -\infty)}$$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{x^2}{x\sqrt{x^2-4x+3}} = \lim_{x\to+\infty} \frac{x}{|x|} = 1$$

$$\lim_{x\to+\infty} f(x) - x = \lim_{x\to+\infty} \frac{x^2}{\sqrt{x^2-4x+3}} - x = \lim_{x\to+\infty} \frac{x^2 - x\sqrt{x^2-4x+3}}{\sqrt{x^2-4x+3}} =$$

$$\lim_{x\to+\infty} \frac{x^4 - x^2(x^2-4x+3)}{\sqrt{x^2-4x+3}(x^2 + x\sqrt{x^2-4x+3})} = \lim_{x\to+\infty} \frac{x^4 - x^4 + 4x^3 - 3x^2}{\sqrt{x^2-4x+3} \cdot (x^2 + x\sqrt{x^2-4x+3})}$$

$$= \lim_{x\to+\infty} \frac{4x^3}{|x| \cdot (x^2 + x|x|)} = \lim_{x\to+\infty} \frac{4x^3}{x \cdot [x^2 + x(x)]} = \lim_{x\to+\infty} \frac{4x^3}{2x^3} = 2$$

$$\Rightarrow \quad \boxed{\text{AO} : y = x + 2 \qquad (x \to +\infty)}$$

Exercice 4.

$\boxed{ED(f) = \mathbb{R} - \{2\}}$

zéro de f : $x^2 - 5x + 8 = 0 \quad \Rightarrow \quad \Delta = 25 - 32 = -7 < 0 \quad \Rightarrow$ pas de zéro

x	$-\infty$	2	$+\infty$
$f(x)$	$-$	||	$+$

$$\lim_{x \to 2} \frac{x^2 - 5x + 8}{x - 2} \overbrace{=}^{"\frac{2}{0}"} \begin{cases} \overset{<}{} & -\infty \\ \underset{>}{} & +\infty \end{cases} \quad \Rightarrow \quad \boxed{\text{AV} : x = 2}$$

$$\frac{x^2 - 5x + 8}{x - 2} = x - 3 + \frac{2}{x - 2} \quad \Rightarrow \quad \delta(x) = \frac{2}{x - 2} \quad \Rightarrow \quad \boxed{\text{AO} : y = x - 3}$$

x	$-\infty$	2	$+\infty$
$\delta(x)$	$-$	||	$+$
position relative	dessous	||	dessus