

Géométrie analytique

Exercice 1.

$$\text{a) } (d) : \begin{cases} x = -2 + 5k \\ y = 3 - 2k \end{cases} \text{ avec } k \in \mathbb{R}$$

$$\text{b) } \vec{n} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} \Rightarrow (d) : 2x + 5y + c = 0$$

$$A \in d \Rightarrow -4 + 15 + c = 0$$

$$\Rightarrow c = -11$$

$$\Rightarrow (d) : 2x + 5y - 11 = 0$$

$$\text{c) } m = -\frac{2}{5}$$

$$(d) : \begin{cases} x = 4 - 2k \\ y = -5 + 7k \end{cases} \text{ avec } k \in \mathbb{R}$$

$$\vec{n} = \begin{pmatrix} 7 \\ 2 \end{pmatrix} \Rightarrow (d) : 7x + 2y + c = 0$$

$$A \in d \Rightarrow 28 - 10 + c = 0$$

$$\Rightarrow c = -18$$

$$\Rightarrow (d) : 7x + 2y - 18 = 0$$

$$m = -\frac{7}{2}$$

Exercice 2.

$$\overrightarrow{AB} = \begin{pmatrix} 6+4 \\ 2+4 \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \end{pmatrix}$$

$$\vec{n}_{AB} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \Rightarrow (AB) : 3x - 5y + c_1 = 0$$

$$A \in AB \Rightarrow -12 + 20 + c_1 = 0$$

$$\Rightarrow c_1 = -8$$

$$\Rightarrow (AB) : 3x - 5y - 8 = 0$$

$$BC \perp AB \Rightarrow (BC) : 5x + 3y + c_2 = 0$$

$$B \in BC \Rightarrow 30 + 6 + c_2 = 0$$

$$\Rightarrow c_2 = -36$$

$$\Rightarrow (BC) : 5x + 3y - 36 = 0$$

$$\overrightarrow{AB} = \begin{pmatrix} 2+4 \\ 4+5 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$$

$$\vec{n}_{AB} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \Rightarrow (AB) : 3x - 2y + c_1 = 0$$

$$A \in AB \Rightarrow -12 + 10 + c_1 = 0$$

$$\Rightarrow c_1 = 2$$

$$\Rightarrow (AB) : 3x - 2y + 2 = 0$$

$$BC \perp AB \Rightarrow (BC) : 2x + 3y + c_2 = 0$$

$$B \in BC \Rightarrow 4 + 12 + c_2 = 0$$

$$\Rightarrow c_2 = -16$$

$$\Rightarrow (BC) : 2x + 3y - 16 = 0$$

$$BC \cap AC : \begin{cases} 5x + 3y = 36 \\ x + 4y = -20 \end{cases}$$

$$\Rightarrow \begin{array}{r} 5x + 3y = 36 \\ -5x - 20y = 100 \\ \hline -17y = 136 \end{array}$$

$$\Rightarrow y = -8, x = 12 \Rightarrow C(12; -8)$$

$$CD // AB \Rightarrow (CD) : 3x - 5y + c_3 = 0$$

$$C \in CD \Rightarrow 36 + 40 + c_3 = 0$$

$$\Rightarrow c_3 = -76$$

$$\Rightarrow (CD) : 3x - 5y - 76 = 0$$

$$AD // BC \Rightarrow (AD) : 5x + 3y + c_4 = 0$$

$$A \in AD \Rightarrow -20 - 12 + c_4 = 0$$

$$\Rightarrow c_4 = 32$$

$$\Rightarrow (AD) : 5x + 3y + 32 = 0$$

$$BC \cap AC : \begin{cases} 2x + 3y = 16 \\ 5x - 12y = 40 \end{cases}$$

$$\Rightarrow \begin{array}{r} 8x + 12y = 64 \\ 5x - 12y = 40 \\ \hline 13x = 104 \end{array}$$

$$\Rightarrow x = 8, y = 0 \Rightarrow C(8; 0)$$

$$CD // AB \Rightarrow (CD) : 3x - 2y + c_3 = 0$$

$$C \in CD \Rightarrow 24 + 0 + c_3 = 0$$

$$\Rightarrow c_3 = -24$$

$$\Rightarrow (CD) : 3x - 2y - 24 = 0$$

$$AD // BC \Rightarrow (AD) : 2x + 3y + c_4 = 0$$

$$A \in AD \Rightarrow -8 - 15 + c_4 = 0$$

$$\Rightarrow c_4 = 23$$

$$\Rightarrow (AD) : 2x + 3y + 23 = 0$$

Exercise 3.

$$a) \overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} -1 - 3 \\ -6 - 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -8 \end{pmatrix}$$

$$\Rightarrow \vec{n}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\Rightarrow (AC) : 2x - y + c_1 = 0$$

$$A \in b \Rightarrow 6 - 2 + c_1 = 0$$

$$\Rightarrow c_1 = -4 \Rightarrow (b) : 2x - y - 4 = 0$$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 4 + 4 \\ -1 + 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

$$\Rightarrow \vec{n}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\Rightarrow (AC) : x - 2y + c_1 = 0$$

$$A \in b \Rightarrow -4 + 10 + c_1 = 0$$

$$\Rightarrow c_1 = -6 \Rightarrow (b) : x - 2y - 6 = 0$$

b) M milieu de AC : $M(1; -2)$

$$\overrightarrow{BM} = \overrightarrow{OM} - \overrightarrow{OB} = \begin{pmatrix} 1+6 \\ -2+1 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$$

$$\Rightarrow \vec{n}_2 = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

$$\Rightarrow (m_b) : x + 7y + c_2 = 0$$

$$B \in m_b \Rightarrow -6 - 7 + c_2 = 0$$

$$\Rightarrow c_2 = 13 \Rightarrow (m_b) : x + 7y + 13 = 0$$

c) $h_b \perp AC \Rightarrow (h_b) : x + 2y + c_3 = 0$

$$B \in h_b \Rightarrow -6 - 2 + c_3 = 0$$

$$\Rightarrow c_3 = 8 \Rightarrow (h_b) : x + 2y + 8 = 0$$

$$d) \delta(B; b) = \frac{|-12 + 1 - 4|}{\sqrt{2^2 + (-1)^2}} = \frac{15}{\sqrt{5}} = 3\sqrt{5} \text{ u}$$

e) $m_{AC} \perp AC \Rightarrow (m_{AC}) : x + 2y + c_4 = 0$

$$M \in m_{AC} \Rightarrow 1 - 4 + c_4 = 0$$

$$\Rightarrow c_4 = 3 \Rightarrow (m_{AC}) : x + 2y + 3 = 0$$

 M milieu de AC : $M(0; -3)$

$$\overrightarrow{BM} = \overrightarrow{OM} - \overrightarrow{OB} = \begin{pmatrix} 0+5 \\ -3-2 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$$

$$\Rightarrow \vec{n}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow (m_b) : x + y + c_2 = 0$$

$$B \in m_b \Rightarrow -5 + 2 + c_2 = 0$$

$$\Rightarrow c_2 = 3 \Rightarrow (m_b) : x + y + 3 = 0$$

 $h_b \perp AC \Rightarrow (h_b) : 2x + y + c_3 = 0$

$$B \in h_b \Rightarrow -10 + 2 + c_3 = 0$$

$$\Rightarrow c_3 = 8 \Rightarrow (h_b) : 2x + y + 8 = 0$$

$$\delta(B; b) = \frac{|-5 - 4 - 6|}{\sqrt{1^2 + (-2)^2}} = \frac{15}{\sqrt{5}} = 3\sqrt{5} \text{ u}$$

 $m_{AC} \perp AC \Rightarrow (m_{AC}) : 2x + y + c_4 = 0$

$$M \in m_{AC} \Rightarrow 0 - 3 + c_4 = 0$$

$$\Rightarrow c_4 = 3 \Rightarrow (m_{AC}) : 2x + y + 3 = 0$$