Limites

Exercice 1.

a) $x^2 - 3 > 0$

x	$-\infty$		$-\sqrt{3}$		$\sqrt{3}$		$+\infty$
x^2-3		$+$	0	$-$	0	$+$	

$\Rightarrow$ $\boxed{ED(f) =]-\infty; -\sqrt{3}[\cup]\sqrt{3}; +\infty[}$

b) zéro : $x^2 - 3 = 1 \quad \Leftrightarrow \quad x^2 - 4 = 0$

$\Rightarrow \quad x = \pm 2$

x	$-\infty$		-2		$-\sqrt{3}$		$\sqrt{3}$		2		$+\infty$
$f(x)$		$+$	0	$-$		▒		$-$	0	$+$	

$x^2 - 15 > 0$

x	$-\infty$		$-\sqrt{15}$		$\sqrt{15}$		$+\infty$
x^2-15		$+$	0	$-$	0	$+$	

$\Rightarrow$ $\boxed{ED(f) =]-\infty; -\sqrt{15}[\cup]\sqrt{15}; +\infty[}$

zéro : $x^2 - 15 = 1 \quad \Leftrightarrow \quad x^2 - 16 = 0$

$\Rightarrow \quad x = \pm 4$

x	$-\infty$		-4		$-\sqrt{15}$		$\sqrt{15}$		4		$+\infty$
$f(x)$		$+$	0	$-$		▒		$-$	0	$+$	

Exercice 2.

a) $$\lim_{x\to-\infty} \frac{3x^2-5x}{4-x} = \lim_{x\to-\infty} \frac{3x^2}{-x}$$

$$= \lim_{x\to-\infty} -3x = \boxed{+\infty}$$

$$\lim_{x\to-\infty} \frac{4x^2-5x}{3-x} = \lim_{x\to-\infty} \frac{4x^2}{-x}$$

$$= \lim_{x\to-\infty} -4x = \boxed{+\infty}$$

b) $$\lim_{\substack{x\to-4 \\ <}} \frac{-4x}{x+4} \overset{\text{"}\frac{16}{0_-}\text{"}}{\overbrace{=}} \boxed{-\infty}$$

$$\lim_{\substack{x\to-5 \\ >}} \frac{7x}{x+5} \overset{\text{"}\frac{-35}{0_+}\text{"}}{\overbrace{=}} \boxed{-\infty}$$

c) $$\lim_{x\to+\infty} \frac{x^3-8x^5-72x^2}{32x^6+7} = \lim_{x\to+-\infty} \frac{-8x^5}{3x^6}$$

$$= \lim_{x\to+\infty} \frac{-8}{3x} = \boxed{0}$$

$$\lim_{x\to+\infty} \frac{5x^6-4000}{4x^4-9x^7-8x^2} = \lim_{x\to+-\infty} \frac{5x^6}{-9x^7}$$

$$= \lim_{x\to+\infty} \frac{5}{-9x} = \boxed{0}$$

d) $\lim\limits_{x\to-4} \dfrac{x^2+5x+4}{x+4} \overbrace{=}^{\text{"}\frac{0}{0}\text{" f.i.}} \lim\limits_{x\to-3} \dfrac{(x+4)(x+1)}{x+4}$

$= \lim\limits_{x\to-4} \; x+1 = \boxed{-3}$

$\lim\limits_{x\to6} \dfrac{x^2-7x+6}{x-6} \overbrace{=}^{\text{"}\frac{0}{0}\text{" f.i.}} \lim\limits_{x\to6} \dfrac{(x-1)(x-6)}{x-6}$

$= \lim\limits_{x\to6} \; x-1 = \boxed{5}$

e) $\lim\limits_{x\to3} \dfrac{\sqrt{x+13}-4}{x-3} \overbrace{=}^{\text{"}\frac{0}{0}\text{" f.i.}}$

$\lim\limits_{x\to3} \dfrac{x-3}{(x-3)(\sqrt{x+13}+4)} =$

$\lim\limits_{x\to3} \dfrac{1}{\sqrt{x+13}+4} = \boxed{\dfrac{1}{8}}$

$\lim\limits_{x\to5} \dfrac{\sqrt{x+20}-5}{x-5} \overbrace{=}^{\text{"}\frac{0}{0}\text{" f.i.}}$

$\lim\limits_{x\to5} \dfrac{x-5}{(x-5)(\sqrt{x+20}+5)} =$

$\lim\limits_{x\to5} \dfrac{1}{\sqrt{x+20}+5} = \boxed{\dfrac{1}{10}}$

f) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{25x^2}}{9-3x} = \lim\limits_{x\to-\infty} \dfrac{5\overbrace{|x|}^{-x}}{-3x} = \boxed{\dfrac{5}{3}}$

$\lim\limits_{x\to-\infty} \dfrac{\sqrt{49x^2}}{8-5x} = \lim\limits_{x\to-\infty} \dfrac{7\overbrace{|x|}^{-x}}{-5x} = \boxed{\dfrac{7}{5}}$

Exercice 3.

a) $f(x) = \dfrac{x^2(x-2)}{(x+2)(x-3)}$

$\boxed{ED(f) = \mathbb{R} - \{-2;3\}}$

b) zéros : $x=0,\ x=2$

x	$-\infty$	-2		0		2		3		$+\infty$
$f(x)$	$-$	$\Vert$	$+$	0	$+$	0	$-$	$\Vert$	$+$	

c) $\lim\limits_{\substack{x\to-2\\<}} f(x) \overbrace{=}^{\text{"}\frac{-16}{0_+}\text{"}} -\infty$ et $\lim\limits_{\substack{x\to-2\\>}} f(x) \overbrace{=}^{\text{"}\frac{-16}{0_-}\text{"}} +\infty$

$\lim\limits_{\substack{x\to3\\<}} f(x) \overbrace{=}^{\text{"}\frac{9}{0_-}\text{"}} -\infty$ et $\lim\limits_{\substack{x\to3\\>}} f(x) \overbrace{=}^{\text{"}\frac{9}{0_+}\text{"}} +\infty$

$\Rightarrow \boxed{\text{AV : } x=-2 \text{ et } x=3}$

$f(x) = \dfrac{x^2(x-3)}{(x+3)(x-5)}$

$\boxed{ED(f) = \mathbb{R} - \{-3;5\}}$

zéros : $x=0,\ x=3$

x	$-\infty$	-3		0		3		5		$+\infty$
$f(x)$	$-$	$\Vert$	$+$	0	$+$	0	$-$	$\Vert$	$+$	

$\lim\limits_{\substack{x\to-3\\<}} f(x) \overbrace{=}^{\text{"}\frac{-54}{0_+}\text{"}} -\infty$ et $\lim\limits_{\substack{x\to-3\\>}} f(x) \overbrace{=}^{\text{"}\frac{-54}{0_-}\text{"}} +\infty$

$\lim\limits_{\substack{x\to5\\<}} f(x) \overbrace{=}^{\text{"}\frac{50}{0_-}\text{"}} -\infty$ et $\lim\limits_{\substack{x\to5\\>}} f(x) \overbrace{=}^{\text{"}\frac{50}{0_+}\text{"}} +\infty$

$\Rightarrow \boxed{\text{AV : } x=-3 \text{ et } x=5}$

$\lim_{x \to \pm\infty} f(x) = \pm\infty \quad \Rightarrow$ pas d'AH

AO $\Rightarrow$ division polynomiale

$$
\begin{array}{l}
x^3 - 2x^2 = \left(x^2 - x - 6\right)\left(x - 1\right) + 5x - 6 \\
-x^3 + x^2 + 6x \\
\hline
-x^2 + 6x \\
x^2 - x - 6 \\
\hline
5x - 6
\end{array}
$$

$\Rightarrow$ $\boxed{\text{AO} : y = x - 1}$

étude du signe de $\delta(x) = \dfrac{5x-6}{x^2-x-6}$

x	$-\infty$	-2	$\frac{6}{5}$	3	$+\infty$
$\delta(x)$	$-$	‖ $+$	0 $-$	‖ $+$	

f est dessus l'AO si $x \in]-2; \frac{6}{5}[\cup]3; +\infty[$

f est dessous l'AO si $x \in]-\infty; -2[\cup]\frac{6}{5}; 3[$

la courbe coupe l'AO en $\left(\frac{6}{5}; \frac{1}{5}\right)$

$\lim_{x \to \pm\infty} f(x) = \pm\infty \quad \Rightarrow$ pas d'AH

AO $\Rightarrow$ division polynomiale

$$
\begin{array}{l}
x^3 - 3x^2 = \left(x^2 - 2x - 15\right)\left(x - 1\right) + 13x - 15 \\
-x^3 + 2x^2 + 15x \\
\hline
-x^2 + 15x \\
x^2 - 2x - 15 \\
\hline
13x - 15
\end{array}
$$

$\Rightarrow$ $\boxed{\text{AO} : y = x - 1}$

étude du signe de $\delta(x) = \dfrac{13x-15}{x^2-2x-15}$

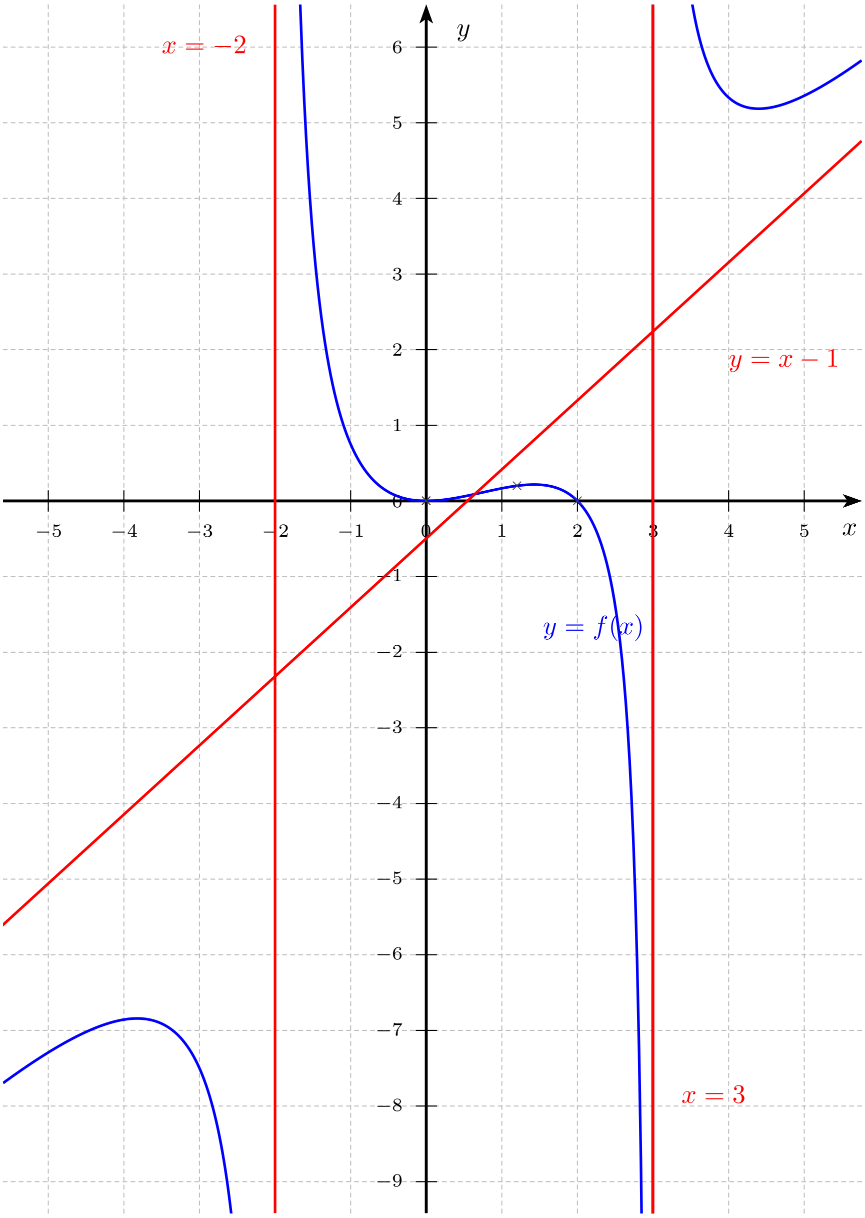
x	$-\infty$	-3	$\frac{15}{13}$	5	$+\infty$
$\cdots$	$-$	‖ $+$	0 $-$	‖ $+$	

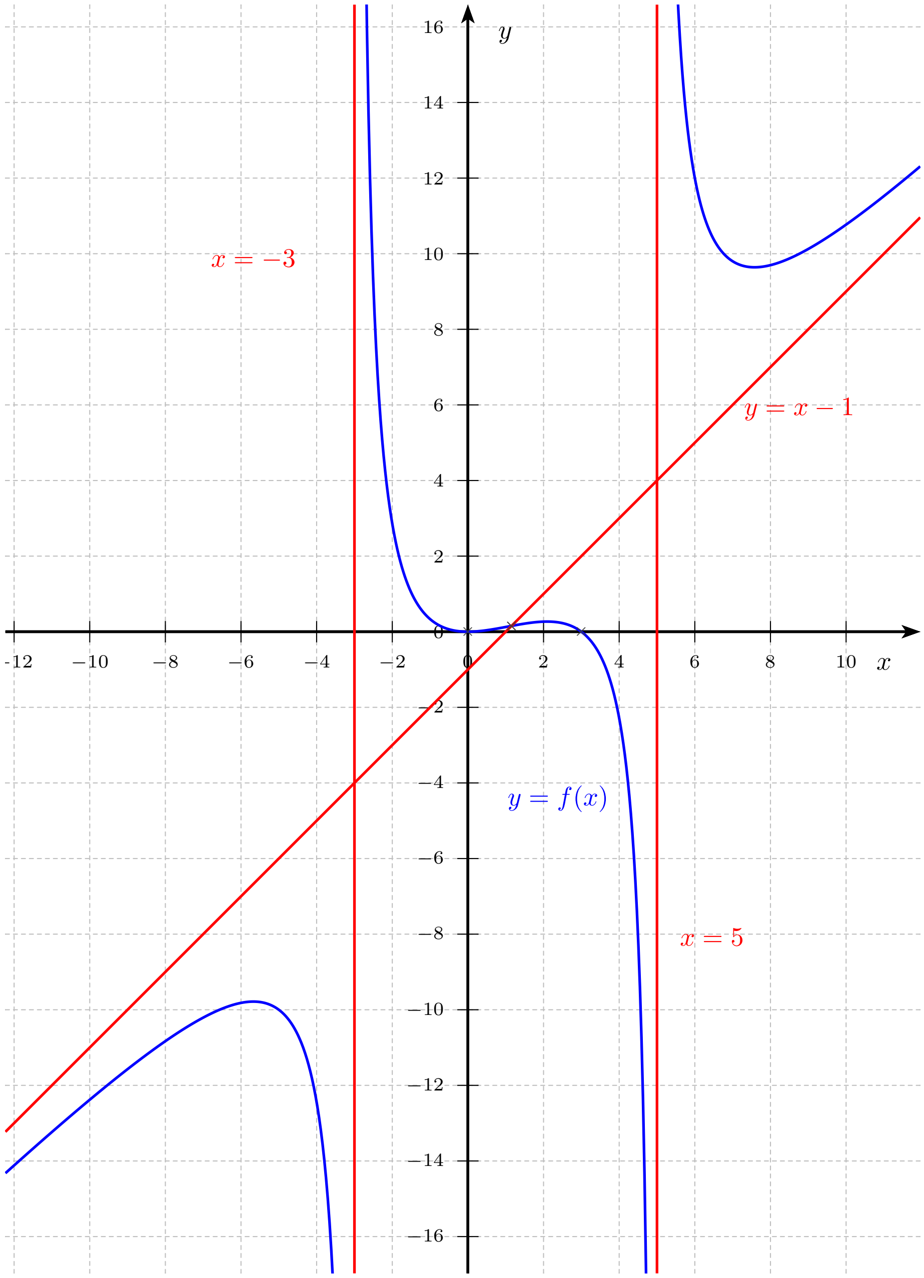
f est dessus l'AO si $x \in]-3; \frac{15}{13}[\cup]5; +\infty[$

f est dessous l'AO si $x \in]-\infty; -3[\cup]\frac{15}{13}; 5[$

la courbe coupe l'AO en $\left(\frac{15}{13}; \frac{2}{13}\right)$

d)




y
x
16
14
12
10
8
6
4
2
0
−2
−4
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−12
−14
−16
-12
−10
−8
−6
−4
−2
0
2
4
6
8
10
$x = -3$
$y = x - 1$
$y = f(x)$
$x = 5$