1.3.1

a) $F(x) = \frac{-2}{\sqrt{x^7}}$ $\qquad f(x) = \frac{7}{\sqrt{x^9}}$

$$F(x) = \frac{-2}{x^{7/2}} = -2\,x^{-7/2} \qquad (x^n)' = n \cdot x^{n-1}$$

$$F'(x) = -2\left(-\frac{7}{2}\right) x^{-7/2-1} = 7\,x^{-9/2} = \frac{7}{x^{9/2}} = \frac{7}{\sqrt{x^9}}$$

b) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$F(x) = \frac{2x^2-1}{2-x^2} + 7$ $\qquad f(x) = \frac{6x}{(2-x^2)^2}$

$$F'(x) = \frac{4x\,(2-x^2) - (2x^2-1)(-2x)}{(2-x^2)^2} = \frac{8x - \cancel{4x^3} + \cancel{4x^3} - 2x}{(2-x^2)^2}$$

$$= \frac{6x}{(2-x^2)^2}$$

c) $F(x) = \frac{2x}{\sqrt{x+1}}$ $\qquad f(x) = \frac{x+2}{\sqrt{(x+1)^3}}$

$$F'(x) = \frac{2\sqrt{x+1} - \cancel{2}x\,\frac{1}{\cancel{2}}(x+1)^{-1/2}}{x+1} = \frac{2\sqrt{x+1} - \frac{x}{\sqrt{x+1}}}{x+1}$$

$$= \frac{\frac{2(x+1)-x}{\sqrt{x+1}}}{x+1} = \frac{2x+2-x}{(x+1)\sqrt{x+1}} = \frac{x+2}{\sqrt{(x+1)^3}}$$

Quelques primitives des fonctions élémentaires

$f(x)$	$\int f(x)\,dx$
1	$x + c \quad (c \in \mathbb{R})$
$x^n \quad (n \neq -1)$	$\frac{1}{n+1}\,x^{n+1} + c \quad (c \in \mathbb{R})$
$\frac{1}{\sqrt{x}}$	$2\sqrt{x} + c \quad (c \in \mathbb{R})$
$\cos(x)$	$\sin(x) + c \quad (c \in \mathbb{R})$
$\sin(x)$	$-\cos(x) + c \quad (c \in \mathbb{R})$

Théorème

Soit f et g des fonctions dérivables sur un intervalle I.

1) $\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

2) $\int k\,f(x)\,dx = k\int f(x)\,dx$ avec $k \in \mathbb{R}$

Exemples

a) $\int (x^2+1)\,dx = \frac{1}{3}x^3 + x + c \quad (c \in \mathbb{R})$

b) $\int \sqrt{x^3}\,dx = \frac{1}{\frac{3}{2}+1}\,x^{\frac{3}{2}+1} + c = \frac{2}{5}\,x^{5/2} + c = \frac{2}{5}\sqrt{x^5} + c$ $(c \in \mathbb{R})$

c) $\int (3x^4 + 5x^3 - 7x^2 + 8x - 4)\,dx$

$$= 3 \cdot \frac{1}{5}x^5 + 5 \cdot \frac{1}{4}x^4 - 7 \cdot \frac{1}{3}x^3 + 8 \cdot \frac{1}{2}x^2 - 4x + c$$

$$= \frac{3}{5}x^5 + \frac{5}{4}x^4 - \frac{7}{3}x^3 + 4x^2 - 4x + c \qquad (c \in \mathbb{R})$$

d) $\int 6\cos(x)\,dx = 6\sin(x) + c \quad (c \in \mathbb{R})$

$$\int x^n\,dx = \frac{1}{n+1}\,x^{n+1} + c$$

1.3.3

a) $\int 3\,dx = 3x + c \quad (c \in \mathbb{R})$

b) $\int 5x \, dx = 5 \frac{1}{2} x^2 + c = \frac{5}{2} x^2 + c \quad (c \in \mathbb{R})$

c) $\int (2x+1) \, dx = x^2 + x + c \quad (c \in \mathbb{R})$

d) $\int (5x-4) \, dx = \frac{5}{2} x^2 - 4x + c \quad (c \in \mathbb{R})$

e) $\int (2x^2 - 3x + 2) \, dx = \frac{2}{3} x^3 - \frac{3}{2} x^2 + 2x + c \quad (c \in \mathbb{R})$

f) $\int 5x^3 \, dx = \frac{5}{4} x^4 + c \quad (c \in \mathbb{R})$

g) $\int (-3x^4) \, dx = -\frac{3}{5} x^5 + c \quad (c \in \mathbb{R})$

h) $\int (3x^5 + 2x^4 - 1) \, dx = \frac{1}{2} x^6 + \frac{2}{5} x^5 - x + c \quad (c \in \mathbb{R})$

i) $\int (\cos(x) + \sin(x)) \, dx = \sin(x) - \cos(x) + c \quad (c \in \mathbb{R})$

j) $\int (1 + \tan^2(x)) \, dx = \tan(x) + c \quad (c \in \mathbb{R})$

1.34

a) $\int \frac{dx}{x^2} = \int \frac{1}{x^2} \, dx = \int x^{-2} \, dx = \frac{1}{(-1)} x^{-1} + c = -\frac{1}{x} + c$

$(c \in \mathbb{R})$

Quelques primitives des fonctions composées

Fonction composée	Primitive	
$u^n(x) \cdot u'(x)$	$\frac{1}{n+1} u^{n+1}(x) + c$	$(n \neq -1$ et $c \in \mathbb{R})$
$\frac{u'(x)}{\sqrt{u(x)}}$	$2\sqrt{u(x)} + c$	$(c \in \mathbb{R})$
$\cos(u(x))\, u'(x)$	$\sin(u(x)) + c$	$(c \in \mathbb{R})$
$\sin(u(x))\, u'(x)$	$\cos(u(x)) + c$	$(c \in \mathbb{R})$

Exemples :

a) $\int 2x \, (x^2+1)^7 \, dx = \frac{1}{8} (x^2+1)^8 + c \quad (c \in \mathbb{R})$

$u = x^2 + 1 \qquad u' = 2x$

$n = 7$

b) $\frac{1}{3} \int 3 \cdot x^2 \, (x^3 - 4)^5 \, dx = \frac{1}{3} \cdot \frac{1}{6} (x^3-4)^6 + c = \frac{1}{18} (x^3-4)^6 + c$

$(c \in \mathbb{R})$

$u = x^3 - 4 \qquad u' = 3x^2$

$n = 5$

c) $\int (x^4+1)^2 \, dx = \int (x^8 + 2x^4 + 1) \, dx = \frac{1}{9} x^9 + \frac{2}{5} x^5 + x + c$

$(c \in \mathbb{R})$

$u = x^4 + 1 \qquad u' = 4x^3$

$\boxed{n = 2}$

d) $\frac{1}{8} \int 8 \cdot \sin(8x) \, dx = -\frac{1}{8} \cos(8x) + c \quad (c \in \mathbb{R})$

$u = 8x \qquad u' = 8$

1.34 pour mercredi

1.34

a) $\int \frac{dx}{x^2} = \int x^{-2}\,dx = -\frac{1}{x} + c \quad (c \in \mathbb{R})$

b) $\int \frac{2\,dx}{x^3} = \int 2x^{-3}dx = 2\,\frac{1}{(-2)}\,x^{-2} + c$

$= -\frac{1}{x^2} + c \quad (c \in \mathbb{R})$

c) $\int -\frac{7}{x^5}\,dx = \int -7x^{-5}dx = -7\,\frac{1}{(-4)}\,x^{-4} + c$

$= \frac{7}{4x^4} + c \quad (c \in \mathbb{R})$

d) $\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{2}{3}\,x^{3/2} + c = \frac{2}{3}\sqrt{x^3} + c$

$(c \in \mathbb{R})$

e) $\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx = \frac{3}{4}\,x^{4/3} + c = \frac{3}{4}\sqrt[3]{x^4} + c$

$(c \in \mathbb{R})$

f) $\int \frac{dx}{\sqrt{x}} = \int x^{-1/2}\,dx = 2\,x^{1/2} + c = 2\sqrt{x} + c$

$(c \in \mathbb{R})$

g) $\int \frac{dx}{\sqrt[3]{x^2}} = \int x^{-2/3}dx = 3\,x^{1/3} + c = 3\sqrt[3]{x} + c$

$(c \in \mathbb{R})$

h) $\int \left(\frac{3}{x^4} - \sqrt[4]{x^3}\right) dx = \int (3x^{-4} - x^{3/4})dx = 3\,\frac{1}{(-3)}\,x^{-3} - \frac{4}{7}\,x^{7/4} + c$

$= -\frac{1}{x^3} - \frac{4}{7}\sqrt[4]{x^7} + c$

1.35

a) $\frac{1}{3}\int 3\cdot\cos(\underbrace{3x}_{u})\,dx = \frac{1}{3}\sin(3x) + c \quad (c \in \mathbb{R})$

$u = 3x \qquad u' = 3$

b) $\frac{1}{2}\int 2\cdot\sin(\underbrace{2x - \frac{\pi}{3}}_{u})\,dx = -\frac{1}{2}\cos(2x - \frac{\pi}{3}) + c$

$(c \in \mathbb{R})$

$u = 2x - \frac{\pi}{3} \qquad u' = 2$

c) $\int 1\cdot(\underbrace{x+3}_{u})^3\,dx = \frac{1}{4}(x+3)^4 + c$

$(c \in \mathbb{R})$

$u = x+3 \qquad u' = 1$

$\int u^n\cdot u'\,dx = \frac{1}{n+1}\,u^{n+1} + c$

f) $\int (\underbrace{3x^2+x}_{u})^3\,(\underbrace{6x+1}_{u'})\,dx = \frac{1}{4}(3x^2+x)^4 + c$

$(c \in \mathbb{R})$

$u = 3x^2 + x \qquad u' = 6x+1$

$\int \sin^2(x)\cos(x)\,dx = \int (\underbrace{\sin(x)}_{u})^2\cdot\cos(x)\,dx$

d) $\frac{1}{2}\int 2\cdot(2x-1)^2\,dx = \frac{1}{2}\cdot\frac{1}{3}(2x-1)^3+C = \frac{1}{6}(2x-1)^3+C$ $(C\in\mathbb{R})$

$u=2x-1 \quad u'=2$

e) $\frac{1}{7}\int 7\cdot(7x-2)^5\,dx = \frac{1}{7}\cdot\frac{1}{6}(7x-2)^6+C = \frac{1}{42}(7x-2)^6+C$ $(C\in\mathbb{R})$

$u=7x-2 \quad u'=7$

f) $\int \underbrace{(3x^2+x)}_{u}{}^3\,\underbrace{(6x+1)}_{u'}\,dx = \frac{1}{4}(3x^2+x)^4+C$ $(C\in\mathbb{R})$

$u=3x^2+x \Rightarrow u'=6x+1$

g) $\frac{1}{8}\int 8\,\underbrace{(4x^2+3)}_{u}{}^4\,x\,dx = \frac{1}{8}\cdot\frac{1}{5}(4x^2+3)^5+C = \frac{1}{40}(4x^2+3)^5+C$ $(C\in\mathbb{R})$

$u=4x^2+3 \Rightarrow u'=8x$

h) $\int \underbrace{\sin^2(x)}_{u}\,\underbrace{\cos(x)}_{u'}\,dx = \frac{1}{3}\sin^3(x)+C$ $(C\in\mathbb{R})$

$u=\sin(x) \Rightarrow u'=\cos(x)$

i) $\int \frac{\overbrace{\tan^2(x)}^{u^2}}{\cos^2(x)}\,dx = \int \underbrace{\frac{1}{\cos^2(x)}}_{u'}\,\underbrace{\tan^2(x)}_{u}\,dx = \frac{1}{3}\tan^3(x)+C$ $(C\in\mathbb{R})$

$u=\tan(x) \Rightarrow u'=\frac{1}{\cos^2(x)}$

j) $\int \sqrt{x+3}\,dx = \int \underbrace{1}_{u'}\cdot\underbrace{(x+3)}_{u}{}^{1/2}\,dx = \frac{2}{3}(x+3)^{3/2}+C = \frac{2}{3}\sqrt{(x+3)^3}+C$ $(C\in\mathbb{R})$

$u=x+3 \Rightarrow u'=1$

k) $\int \frac{dx}{\sqrt{3x+1}} = \frac{1}{3}\int 3\cdot\underbrace{(3x+1)}_{u}{}^{-1/2}\,dx = \frac{1}{3}\cdot 2\,(3x+1)^{1/2}+C$

$= \frac{2}{3}\sqrt{3x+1}+C$ $(C\in\mathbb{R})$

$u=3x+1 \Rightarrow u'=3$

l) $\int \frac{x+1}{\sqrt{x^2+2x}}\,dx = \frac{1}{2}\int 2\cdot(x+1)\,\underbrace{(x^2+2x)}_{u}{}^{-1/2}\,dx = \frac{1}{2}\cdot 2\,(x^2+2x)^{1/2}+C$

$= \sqrt{x^2+2x}+C$ $(C\in\mathbb{R})$

$u=x^2+2x \Rightarrow u'=2x+2=2(x+1)$

1.3.6

a) $\int (3x^2-2x+3)\,dx = 3\cdot\frac{1}{3}x^3-2\cdot\frac{1}{2}x^2+3x+C$

$= x^3-x^2+3x+C$ $(C\in\mathbb{R})$

b) $\int \frac{3x^4-3x^2-7}{4x^2}\,dx = \int\left(\frac{3x^{\cancel{4}\,2}}{4\cancel{x^2}}-\frac{3\cancel{x^2}}{4\cancel{x^2}}-\frac{7}{4x^2}\right)dx$

$= \int\left(\frac{3}{4}x^2-\frac{3}{4}-\frac{7}{4}x^{-2}\right)dx$

$= \frac{3}{4}\frac{1}{3}x^3-\frac{3}{4}x-\frac{7}{4}\frac{1}{(-1)}x^{-1}+C = \frac{1}{4}x^3-\frac{3}{4}x+\frac{7}{4x}+C$

$(C\in\mathbb{R})$

e) $\int \left(2\sin(x) - 3\cos(x)\right)dx = -2\cos(x) - 3\sin(x) + C \quad (C \in \mathbb{R})$

f) $\frac{1}{2}\int 2 \cdot \cos(\underbrace{2x}_{u})\,dx = \frac{1}{2}\sin(2x) + C \quad (C \in \mathbb{R})$

$u = 2x \qquad u' = 2$

g) $\int \left(\frac{5}{\cos^2(x)} + 5\cos(x)\right)dx = 5\tan(x) + 5\sin(x) + C$

$(C \in \mathbb{R})$

h) $\int \left(8\sin(x) + \frac{4}{\sqrt{2x}}\right)dx = \int \left(8\sin(x) + 4\,(2x)^{-1/2}\right)dx$