Dérivées

Exercice 1.

a) $\boxed{12\,x^3 - 10\,x + 7}$

$\boxed{15\,x^4 - 12\,x^2 + 6}$

b) $\dfrac{4\,x^3}{3\,(x^4+1)^{\frac{2}{3}}} = \boxed{\dfrac{4x^3}{3\sqrt[3]{(x^4+1)^2}}}$

$\dfrac{2\,x}{5\,(x^2+1)^{\frac{4}{5}}} = \boxed{\dfrac{2x}{5\sqrt[5]{(x^2+1)^4}}}$

c) $\boxed{-5\,\sin(5\,x)}$

$\boxed{-7\,\sin(7\,x)}$

d) $\dfrac{(2\,x-4)(x-5)-(x^2-4x+3)\cdot 1}{(x-5)^2}$

$= \dfrac{2x^2-14x+20-x^2+4x-3}{(x-5)^2}$

$= \boxed{\dfrac{x^2-10x+17}{(x-5)^2}}$

$\dfrac{(2\,x-4)(x-8)-(x^2-4x+5)\cdot 1}{(x-8)^2}$

$= \dfrac{2x^2-20x+32-x^2+4x-5}{(x-8)^2}$

$= \boxed{\dfrac{x^2-16x+27}{(x-8)^2}}$

e) $12(3x-2)^3(-4x+3)^3 - 12(3x-2)^4(-4x+3)^2$

$= 12(3x-2)^3(-4x+3)^2[(-4x+3)-(3x-2)]$

$= \boxed{12(3x-2)^3(-4x+3)^2(-7x+5)}$

$10(2x-4)^4(-5x+2)^2 - 10(2x-4)^5(-5x+2)$

$= 10(2x-4)^4(-5x+2)[(-5x+2)-(2x-4)]$

$= \boxed{10(2x-4)^4(-5x+2)(-7x+6)}$

f) $\dfrac{-6}{7}x^{\frac{-13}{7}} = \boxed{\dfrac{-6}{7\sqrt[7]{x^{13}}}}$

$\dfrac{-4}{5}x^{\frac{-9}{5}} = \boxed{\dfrac{-4}{5\sqrt[5]{x^9}}}$

Exercice 2.

$f(6) = 5$

$f'(x) = \dfrac{4}{2\sqrt{4x+1}} \quad \Rightarrow \quad f'(6) = \dfrac{2}{5}$

$(t)\,:\; y - 5 = \dfrac{2}{5}(x-6)$

$\Leftrightarrow \quad y = \dfrac{2}{5}x - \dfrac{12}{5} + 5$

$\Leftrightarrow \boxed{(t)\,:\; y = \dfrac{2}{5}x + \dfrac{13}{5}}$

$f(4) = 5$

$f'(x) = \dfrac{6}{2\sqrt{6x+1}} \quad \Rightarrow \quad f'(4) = \dfrac{3}{5}$

$(t)\,:\; y - 5 = \dfrac{3}{5}(x-4)$

$\Leftrightarrow \quad y = \dfrac{3}{5}x - \dfrac{12}{5} + 5$

$\Leftrightarrow \boxed{(t)\,:\; y = \dfrac{3}{5}x + \dfrac{13}{5}}$

Exercice 3.

$f(-4) = -32$

$f'(x) = 15(3x+10)^4 \quad \Rightarrow \quad f'(-4) = 240$

$(t) : \ y + 32 = 240(x+4)$

$\Leftrightarrow \boxed{(t) : \ y = 240x + 928}$

$f(-5) = -27$

$f'(x) = 6(2x+7)^2 \quad \Rightarrow \quad f'(-5) = 54$

$(t) : \ y + 27 = 54(x+5)$

$\Leftrightarrow \boxed{(t) : \ y = 54x + 243}$