Exponentielles et logarithmes

Exercice 1.

a) $7^{2x-1} = 2401$

$\Leftrightarrow \quad 2x - 1 = \log_7(2401) = 4$

$\Leftrightarrow \quad 2x = 5 \quad \Leftrightarrow \quad x = \dfrac{5}{2}$

$\Rightarrow \boxed{S = \left\{ \dfrac{5}{2} \right\}}$

$4^{5x-1} = 16384$

$\Leftrightarrow \quad 5x - 1 = \log_4(16384) = 7$

$\Leftrightarrow \quad 5x = 8 \quad \Leftrightarrow \quad x = \dfrac{8}{5}$

$\Rightarrow \boxed{S = \left\{ \dfrac{8}{5} \right\}}$

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b) $ED =]2; +\infty[$

$\log_8(x^2 - 4) = \log_8(8) + \log_8(x + 2)$

$\Leftrightarrow \quad \log_8(x^2 - 4) = \log_8(8(x + 2))$

$\Leftrightarrow \quad x^2 - 4 = 8x + 16 \quad \Leftrightarrow \quad x^2 - 8x - 20 = 0$

$\Leftrightarrow \quad (x - 10)(x + 2) = 0$

$\Rightarrow \quad x = 10$ et $x = -2$ (sol. à élim.)

$\Rightarrow \boxed{S = \{10\}}$

$ED =]-\infty; 0[$

$\log_8(x^2 - 6x) = \log_8(8) + \log_8(3 - x)$

$\Leftrightarrow \quad \log_8(x^2 - 6x) = \log_8(8(3 - x))$

$\Leftrightarrow \quad x^2 - 6x = 24 - 8x \Leftrightarrow x^2 + 2x - 24 = 0$

$\Leftrightarrow \quad (x + 6)(x - 4) = 0$

$\Rightarrow \quad x = -6$ et $x = 4$ (sol. à élim.)

$\Rightarrow \boxed{S = \{-6\}}$

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c) $x^2 = 196 \Rightarrow x = \sqrt{196} = 14 \ (-14 \text{ à élim.})$

$\Rightarrow \boxed{S = \{14\}}$

$x^2 = 169 \Rightarrow x = \sqrt{169} = 13 \ (-13 \text{ à élim.})$

$\Rightarrow \boxed{S = \{13\}}$

Exercice 2.

a) $\dfrac{4x-1}{x+5} > 0$

x	$-\infty$	-5		$\frac{1}{4}$	$+\infty$
$\frac{4x-1}{x+5}$		$+$	$\|$	$-$ 0	$+$

$\Rightarrow$ $\boxed{ED(f) =\,]-\infty;-5[\,\cup\,]\frac{1}{4};+\infty[}$

$\dfrac{3x-2}{x+8} > 0$

x	$-\infty$	-8		$\frac{2}{3}$	$+\infty$
$\frac{3x-2}{x+8}$		$+$	$\|$	$-$ 0	$+$

$\Rightarrow$ $\boxed{ED(f) =\,]-\infty;-8[\,\cup\,]\frac{2}{3};+\infty[}$

b) zéro : $\dfrac{4x-1}{x+5} = 1 \quad \Rightarrow \quad 4x-1 = x+5$

$\Rightarrow \quad 3x = 6 \quad \Rightarrow \quad x = 2$

x	$-\infty$	-5		$\frac{1}{4}$		2	$+\infty$
$f(x)$		$+$			$-$	0	$+$

zéro : $\dfrac{3x-2}{x+8} = 1 \quad \Rightarrow \quad 3x-2 = x+8$

$\Rightarrow \quad 2x = 10 \quad \Rightarrow \quad x = 5$

x	$-\infty$	-8		$\frac{2}{3}$		5	$+\infty$
$f(x)$		$+$			$-$	0	$+$

Exercice 3.

a) $N(20) = \dfrac{200}{1+300\cdot e^{-3,2}} \simeq 15.12$

$\Rightarrow$ $\boxed{\text{15 chimpanzés infectés}}$

$N(15) = \dfrac{300}{1+400\cdot e^{-2,7}} \simeq 10.76$

$\Rightarrow$ $\boxed{\text{11 chimpanzés infectés}}$

b) $\dfrac{200}{1+300\cdot e^{-0,16t}} = 120$

$\Rightarrow \quad 200 = 120\left(1+300\cdot e^{-0,16t}\right)$

$\Rightarrow \quad 1+300\cdot e^{-0,16t} = \dfrac{5}{3}$

$\Rightarrow \quad 300\cdot e^{-0,16t} = \dfrac{2}{3} \quad \Rightarrow \quad e^{-0,16t} = \dfrac{2}{900}$

$\Rightarrow \quad -0,16t = \ln\left(\frac{2}{900}\right)$

$\Rightarrow \quad t = -\frac{1}{0,16}\cdot\ln\left(\frac{2}{900}\right) \simeq 38.18$

$\Rightarrow$ $\boxed{\text{dans 39 jours}}$

$\dfrac{300}{1+400\cdot e^{-0,18t}} = 160$

$\Rightarrow \quad 300 = 160\left(1+400\cdot e^{-0,18t}\right)$

$\Rightarrow \quad 1+400\cdot e^{-0,18t} = \dfrac{15}{8}$

$\Rightarrow \quad 400\cdot e^{-0,18t} = \dfrac{7}{8} \quad \Rightarrow \quad e^{-0,18t} = \dfrac{7}{3200}$

$\Rightarrow \quad -0,18t = \ln\left(\frac{7}{3200}\right)$

$\Rightarrow \quad t = -\frac{1}{0,18}\cdot\ln\left(\frac{7}{3200}\right) \simeq 34.03$

$\Rightarrow$ $\boxed{\text{dans 35 jours}}$

Exercice 4.

a) $P(t) = \boxed{50'000 \cdot 2^{\frac{t}{20}}}$

b) $P(30) = 50'000 \cdot 2^{\frac{3}{2}} \simeq \boxed{141'421 \text{ habitants}}$

c) $200'000 = 50'000 \cdot 2^{\frac{t}{20}} \quad \Leftrightarrow \quad 4 = 2^{\frac{t}{20}}$

$\Leftrightarrow t = 20 \cdot \log_2(4) = 40 \Rightarrow \boxed{\text{en l'an 2044}}$

$P(t) = \boxed{40'000 \cdot 3^{\frac{t}{30}}}$

$P(50) = 40'000 \cdot 3^{\frac{5}{3}} \simeq \boxed{249'610 \text{ habitants}}$

$360'000 = 40'000 \cdot 2^{\frac{t}{30}} \quad \Leftrightarrow \quad 9 = 3^{\frac{t}{30}}$

$\Leftrightarrow t = 30 \cdot \log_3(9) = 60 \Rightarrow \boxed{\text{en l'an 2054}}$

Exercice 5.

a) $Q(t) = \boxed{200 \cdot 0,5^{\frac{t}{5}}}$

b) $Q(8) = 200 \cdot 0,5^{\frac{8}{5}} \simeq \boxed{66 \text{ grammes}}$

c) $20 = 200 \cdot 0,5^{\frac{t}{5}} \quad \Leftrightarrow \quad 0,1 = 0,5^{\frac{t}{5}}$

$\Leftrightarrow t = 5 \cdot \log_{0,5}(0,1) \simeq 16.61$

$\Rightarrow \boxed{16 \text{ heures et 37 minutes}}$

$Q(t) = \boxed{250 \cdot 0,5^{\frac{t}{4}}}$

$Q(12) = 250 \cdot 0,5^{\frac{12}{4}} = \boxed{31,25 \text{ grammes}}$

$12,5 = 250 \cdot 0,5^{\frac{t}{4}} \quad \Leftrightarrow \quad 0,05 = 0,5^{\frac{t}{4}}$

$\Leftrightarrow t = 4 \cdot \log_{0,05}(0,5) \simeq 17,29$

$\Rightarrow \boxed{17 \text{ heures et 17 minutes}}$