

Géométrie analytique

Exercice 1.

$$a) \overrightarrow{AB} = \begin{pmatrix} 3+1 \\ 1-5 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$(d) : \begin{cases} x = -1 + k \\ y = 5 - k \end{cases} \text{ avec } k \in \mathbb{R}$$

$$b) \vec{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow (d) : x + y + c = 0$$

$$A \in d \Rightarrow -1 + 5 + c = 0$$

$$\Rightarrow c = -4$$

$$\Rightarrow (d) : x + y - 4 = 0$$

$$c) m = -1$$

$$\overrightarrow{AB} = \begin{pmatrix} 5-2 \\ 2+3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$(d) : \begin{cases} x = 2 + 3k \\ y = -3 + 5k \end{cases} \text{ avec } k \in \mathbb{R}$$

$$\vec{n} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} \Rightarrow (d) : 5x - 3y + c = 0$$

$$A \in d \Rightarrow 10 + 9 + c = 0$$

$$\Rightarrow c = -19$$

$$\Rightarrow (d) : 5x - 3y - 19 = 0$$

$$m = \frac{5}{3}$$

Exercice 2.

$$a) D \in d? : 6 - 4 + 2 = 0 \checkmark$$

$$b) (p) : 4x + 3y + c = 0$$

$$D \in p \Rightarrow 8 + 3 + c = 0 \Leftrightarrow c = -11$$

$$\Rightarrow (p) : 4x + 3y - 11 = 0$$

$$c) (e) : 3x - 4y + c = 0$$

$$\delta(D; e) = \frac{|2 + c|}{5} = 10$$

$$\Rightarrow 2 + c = \pm 50 \Rightarrow c_1 = 48, c_2 = -52$$

$$(e_1) : 3x - 4y + 48 = 0$$

$$(e_2) : 3x - 4y - 52 = 0$$

$$D \in d? : 4 - 9 + 5 = 0 \checkmark$$

$$(p) : 3x - 4y + c = 0$$

$$D \in p \Rightarrow 3 + 12 + c = 0 \Leftrightarrow c = -15$$

$$\Rightarrow (p) : 3x - 4y - 15 = 0$$

$$(e) : 4x + 3y + c = 0$$

$$\delta(D; e) = \frac{|-5 + c|}{5} = 10$$

$$\Rightarrow -5 + c = \pm 50 \Rightarrow c_1 = 55, c_2 = -45$$

$$(e_1) : 4x + 3y + 55 = 0$$

$$(e_2) : 4x + 3y - 45 = 0$$

$$e_1 \cap p : \begin{cases} 3x - 4y = -48 \\ 4x + 3y = 11 \end{cases}$$

$$\Rightarrow \begin{array}{r} 9x - 12y = -144 \\ 16x + 12y = 44 \\ \hline 25x = -100 \end{array}$$

$$\Rightarrow x = -4 \text{ et } y = 9 \Rightarrow A_1(-4; 9)$$

$$e_2 \cap p : \begin{cases} 3x - 4y = 52 \\ 4x + 3y = 11 \end{cases}$$

$$\Rightarrow \begin{array}{r} 9x - 12y = 156 \\ 16x + 12y = 44 \\ \hline 25x = 200 \end{array}$$

$$\Rightarrow x = 8 \text{ et } y = -7 \Rightarrow A_2(8; -7)$$

$$e_1 \cap p : \begin{cases} 4x + 3y = -55 \\ 3x - 4y = 15 \end{cases}$$

$$\Rightarrow \begin{array}{r} 16x + 12y = -220 \\ 9x - 12y = 45 \\ \hline 25x = -175 \end{array}$$

$$\Rightarrow x = -7 \text{ et } y = -9 \Rightarrow A_1(-7; -9)$$

$$e_2 \cap p : \begin{cases} 4x + 3y = 45 \\ 3x - 4y = 15 \end{cases}$$

$$\Rightarrow \begin{array}{r} 16x + 12y = 180 \\ 9x - 12y = 45 \\ \hline 25x = 225 \end{array}$$

$$\Rightarrow x = 9 \text{ et } y = 3 \Rightarrow A_2(9; 3)$$

Exercice 3.

$$\text{a) } \overrightarrow{AC} = \begin{pmatrix} -4 - 2 \\ -5 + 1 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \end{pmatrix} = -2 \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\Rightarrow \vec{n}_1 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\Rightarrow (AC) : 2x - 3y + c_1 = 0$$

$$A \in b \Rightarrow 4 + 3 + c_1 = 0$$

$$\Rightarrow c_1 = -7 \Rightarrow (b) : 2x - 3y - 7 = 0$$

$$\overrightarrow{AC} = \begin{pmatrix} 6 - 2 \\ 3 - 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\Rightarrow \vec{n}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\Rightarrow (AC) : x - 2y + c_1 = 0$$

$$A \in b \Rightarrow 2 - 2 + c_1 = 0$$

$$\Rightarrow c_1 = 0 \Rightarrow (b) : x - 2y = 0$$

b) M milieu de AC : $M(-1; -3)$

$$\overrightarrow{BM} = \begin{pmatrix} -1+1 \\ -3-1 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} = -4 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \vec{n}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow (m_b) : x + c_2 = 0$$

$$B \in m_b \Rightarrow -1 + c_2 = 0$$

$$\Rightarrow c_2 = 1 \Rightarrow (m_b) : x + 1 = 0$$

c) $\overrightarrow{AB} = \begin{pmatrix} -1-2 \\ 1+1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$$\Rightarrow \vec{n}_3 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\Rightarrow (AB) : 2x + 3y + c_3 = 0$$

$$A \in AB \Rightarrow 4 - 3 + c_3 = 0$$

$$\Rightarrow c_3 = -1 \Rightarrow (AB) : 2x + 3y - 1 = 0$$

$$\text{bissectrices : } \frac{2x - 3y - 7}{\sqrt{13}} = \pm \frac{2x + 3y - 1}{\sqrt{13}}$$

$$\oplus -6y - 6 = 0 \Leftrightarrow y + 1 = 0 \checkmark$$

$$\ominus 4x - 8 = 0 \Leftrightarrow x - 2 = 0$$

$$\Rightarrow (b_a) : y + 1 = 0$$

d) $\delta(B; AC) = \frac{|-2-3-7|}{\sqrt{13}} = \frac{12}{\sqrt{13}}$

$$= \frac{12\sqrt{13}}{13} \text{ u}$$

e) $m_{AC} \perp AC \Rightarrow (m_{AC}) : 3x + 2y + c_4 = 0$

$$M \in m_{AC} \Rightarrow -3 - 6 + c_4 = 0$$

$$\Rightarrow c_4 = 9 \Rightarrow (m_{AC}) : 3x + 2y + 9 = 0$$

 M milieu de AC : $M(4; 2)$

$$\overrightarrow{BM} = \begin{pmatrix} 4-8 \\ 2+2 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} = -4 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \vec{n}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow (m_b) : x + y + c_2 = 0$$

$$B \in m_b \Rightarrow 8 - 2 + c_2 = 0$$

$$\Rightarrow c_2 = -6 \Rightarrow (m_b) : x + y - 6 = 0$$

$$\overrightarrow{AB} = \begin{pmatrix} 8-2 \\ -2-1 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\Rightarrow \vec{n}_3 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\Rightarrow (AB) : x + 2y + c_3 = 0$$

$$A \in AB \Rightarrow 2 + 2 + c_3 = 0$$

$$\Rightarrow c_3 = -4 \Rightarrow (AB) : x + 2y - 4 = 0$$

$$\text{bissectrices : } \frac{x - 2y}{\sqrt{5}} = \pm \frac{x + 2y - 4}{\sqrt{5}}$$

$$\oplus -4y + 4 = 0 \Leftrightarrow y - 1 = 0 \checkmark$$

$$\ominus 2x - 4 = 0 \Leftrightarrow x - 2 = 0$$

$$\Rightarrow (b_a) : y - 1 = 0$$

$$\delta(B; AC) = \frac{|8+4|}{\sqrt{5}} = \frac{12}{\sqrt{5}}$$

$$= \frac{12\sqrt{5}}{5} \text{ u}$$

$$m_{AC} \perp AC \Rightarrow (m_{AC}) : 2x + y + c_4 = 0$$

$$M \in m_{AC} \Rightarrow 8 + 2 + c_4 = 0$$

$$\Rightarrow c_4 = -10 \Rightarrow (m_{AC}) : 2x + y - 10 = 0$$