Combinatoire

Exercice 1.

a) $P_{12} = 12! = \boxed{479\ 001\ 600}$

b) $P_2 \cdot P_6 \cdot P_6 = 2! \cdot 6! \cdot 6! = \boxed{1\ 036\ 800}$

c) $C_1^3 \cdot P_4 \cdot P_8 = 3 \cdot 4! \cdot 8! = \boxed{2\ 903\ 040}$

$P_{10} = 10! = \boxed{3\ 628\ 800}$

$P_2 \cdot P_5 \cdot P_5 = 2! \cdot 5! \cdot 5! = \boxed{28\ 800}$

$C_1^5 \cdot P_2 \cdot P_8 = 5 \cdot 2! \cdot 8! = \boxed{403\ 200}$

Exercice 2.

a) $\bar{A}_6^4 = 4^6 = \boxed{4\ 096}$

b) $C_4^6 \cdot \bar{A}_4^1 \cdot \bar{A}_2^3 = 15 \cdot 1 \cdot 9 = \boxed{135}$

c) $4'096 - \bar{A}_6^3 = 4\ 096 - 729 = \boxed{3\ 367}$

$\bar{A}_8^5 = 5^8 = \boxed{390\ 625}$

$C_5^8 \cdot \bar{A}_5^1 \cdot \bar{A}_3^4 = 56 \cdot 1 \cdot 64 = \boxed{3\ 584}$

$390'625 - \bar{A}_8^4 = 390\ 625 - 65\ 536 = \boxed{325\ 089}$

Exercice 3.

a) $C_5^{20} = \boxed{15\ 504}$

b) $C_5^{20} - C_5^{14} = \boxed{13\ 502}$

c) 2 blancs : $C_2^8 \cdot C_1^6 \cdot C_1^4 \cdot C_1^2 = 1\ 344$

2 noirs : $C_1^8 \cdot C_2^6 \cdot C_1^4 \cdot C_1^2 = 960$

2 bruns : $C_1^8 \cdot C_1^6 \cdot C_2^4 \cdot C_1^2 = 576$

2 gris : $C_1^8 \cdot C_1^6 \cdot C_1^4 \cdot C_2^2 = 192$

$\Rightarrow \boxed{3\ 072}$

$C_5^{21} = \boxed{20\ 349}$

$C_5^{21} - C_5^{14} = \boxed{18\ 347}$

2 blancs : $C_2^9 \cdot C_1^5 \cdot C_1^4 \cdot C_1^3 = 2\ 160$

2 noirs : $C_1^9 \cdot C_2^5 \cdot C_1^4 \cdot C_1^3 = 1\ 080$

2 bruns : $C_1^9 \cdot C_1^5 \cdot C_2^4 \cdot C_1^3 = 810$

2 gris : $C_1^9 \cdot C_1^5 \cdot C_1^4 \cdot C_2^3 = 540$

$\Rightarrow \boxed{4\ 590}$

Exercice 4.

$\bar{P}_5(5) + \bar{P}_5(4) + \bar{P}_5(3,2) = 1 + 5 + 10 = \boxed{16}$ | $\bar{P}_6(6) + \bar{P}_6(5) + \bar{P}_6(4,2) = 1 + 6 + 15 = \boxed{22}$