Géométrie analytique II et intégrales indéfinies

Exercice 1.

a) $x^2 - 2x + 1 + y^2 - 16y + 64 = -7 + 1 + 64$

$\Rightarrow \quad (x-1)^2 + (y-8)^2 = 58$

$\Rightarrow \boxed{C(1;8) \text{ et } r = \sqrt{58} \text{ u}}$

b) $\overrightarrow{AC} = \begin{pmatrix} 1+3 \\ 8+2 \end{pmatrix} = \begin{pmatrix} 4 \\ 10 \end{pmatrix} = 2 \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$\Rightarrow \quad (m) : 2x + 5y + c = 0$

M milieu de AC :

$M\left(\dfrac{-3+1}{2}; \dfrac{-2+8}{2}\right) \Rightarrow \quad M(-1;3)$

$M \in m \Rightarrow -2 + 15 + c = 0 \quad \Rightarrow \quad c = -13$

$\Rightarrow \boxed{(m) \ : \ 2x + 5y - 13 = 0}$

c) $x = \dfrac{-5y+13}{2}$

$\Rightarrow \left(\dfrac{-5y+13}{2} - 1\right)^2 + (y-8)^2 = 58$

$\Rightarrow \left(\dfrac{-5y+11}{2}\right)^2 + (y-8)^2 = 58$

$\Rightarrow \dfrac{25y^2 - 110y + 121}{4} + y^2 - 16y + 64 = 58$

$\Rightarrow \quad 25y^2 - 110y + 121 + 4y^2 - 64y + 24 = 0$

$\Rightarrow \quad 29y^2 - 174y + 145 = 0$

$\Rightarrow \quad 29(y^2 - 6y + 5) = 0$

$\Rightarrow \quad 29(y-5)(y-1) = 0$

$\Rightarrow \quad y_1 = 5 \text{ et } y_2 = 1 \Rightarrow x_1 = -6 \text{ et } x_2 = 4$

$x^2 - 4x + 4 + y^2 - 14y + 49 = -12 + 4 + 49$

$\Rightarrow \quad (x-2)^2 + (y-7)^2 = 41$

$\Rightarrow \boxed{C(2;7) \text{ et } r = \sqrt{41} \text{ u}}$

$\overrightarrow{AC} = \begin{pmatrix} 2-1 \\ 7+2 \end{pmatrix} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$

$\Rightarrow \quad (m) : x + 9y + c = 0$

M milieu de AC :

$M\left(\dfrac{1+2}{2}; \dfrac{-2+7}{2}\right) \Rightarrow \quad M(\frac{3}{2}; \frac{5}{2})$

$M \in m \Rightarrow \dfrac{3}{2} + \dfrac{45}{2} + c = 0 \quad \Rightarrow \quad c = -24$

$\Rightarrow \boxed{(m) \ : \ x + 9y - 24 = 0}$

$x = -9y + 24$

$\Rightarrow (-9y + 24 - 2)^2 + (y-7)^2 = 41$

$\Rightarrow (-9y + 22)^2 + (y-7)^2 = 41$

$\Rightarrow 81y^2 - 396y + 484 + y^2 - 14y + 49 = 41$

$\Rightarrow \quad 82y^2 - 410y + 492 = 0$

$\Rightarrow \quad 82(y^2 - 5y + 6) = 0$

$\Rightarrow \quad 82(y-2)(y-3) = 0$

$\Rightarrow \quad y_1 = 2 \text{ et } y_2 = 3 \Rightarrow x_1 = 6 \text{ et } x_2 = -3$

$\Rightarrow \boxed{D(-3;3) \text{ et } B(6;2)}$

$\Rightarrow \boxed{D(-6;5) \text{ et } B(4;1)}$

d) équ. dédoublée de γ

$(x-1)(x-1)+(y-8)(y-8)=58$

$\Rightarrow \quad -7(x-1)-3(y-8)=58$

$\Rightarrow \quad -7x+7-3y+24=58$

$\Rightarrow \boxed{(t_1): 7x+3y+27=0}$

e) $A \in t_1 : -21-6+27=0$ ✓

$(t): y=mx+h \Rightarrow \quad mx-y+h=0$

$A \in t \Rightarrow -2=-3m+h \Rightarrow h=3m-2$

$\Rightarrow \quad (t): mx-y+3m-2=0$

$\delta(C;t)=\sqrt{58}$

$$\Rightarrow \quad \frac{|m-8+3m-2|}{\sqrt{m^2+1}}=\sqrt{58}$$

$\Rightarrow \quad 4m-10=\pm\sqrt{58}\sqrt{m^2+1}$

$\Rightarrow \quad (4m-10)^2=58(m^2+1)$

$\Rightarrow \quad 16m^2-80m+100=58m^2+58$

$\Rightarrow \quad 2(21m^2+40m-21)=0$

$\Rightarrow \quad 2(3m+7)(7m-3)=0$

$$\Rightarrow \quad m_1=-\frac{7}{3} \text{ et } m_2=\frac{3}{7}$$

$$\Rightarrow \quad h_1=-7-2=-9 \text{ et } h_2=\frac{9}{7}-2=-\frac{5}{7}$$

$$\Rightarrow \quad y=-\frac{7}{3}x-9 \Rightarrow (t_1): 7x+3y+27=0$$

$$\Rightarrow y=\frac{3}{7}x-\frac{5}{7} \Rightarrow \boxed{(t_2): 3x-7y-5=0}$$

équ. dédoublée de γ

$(x-2)(x-2)+(y-7)(y-7)=41$

$\Rightarrow \quad -5(x-2)-4(y-7)=41$

$\Rightarrow \quad -5x+10-4y+28=41$

$\Rightarrow \boxed{(t_1): 5x+4y+3=0}$

$A \in t_1 : 5-8+3=0$ ✓

$(t): y=mx+h \Rightarrow \quad mx-y+h=0$

$A \in t \Rightarrow -2=m+h \Rightarrow h=-m-2$

$\Rightarrow \quad (t): mx-y-m-2=0$

$\delta(C;t)=\sqrt{41}$

$$\Rightarrow \quad \frac{|2m-7-m-2|}{\sqrt{m^2+1}}=\sqrt{41}$$

$\Rightarrow \quad m-9=\pm\sqrt{41}\sqrt{m^2+1}$

$\Rightarrow \quad (m-9)^2=41(m^2+1)$

$\Rightarrow \quad m^2-18m+81=41m^2+41$

$\Rightarrow \quad 2(20m^2+9m-20)=0$

$\Rightarrow \quad 2(4m+5)(5m-4)=0$

$$\Rightarrow \quad m_1=-\frac{5}{4} \text{ et } m_2=\frac{4}{5}$$

$$\Rightarrow \quad h_1=\frac{5}{4}-2=-\frac{3}{4} \text{ et } h_2=-\frac{4}{5}-2=-\frac{14}{5}$$

$$\Rightarrow \quad y=-\frac{5}{4}x-\frac{3}{4} \Rightarrow (t_1): 5x+4y+3=0$$

$$\Rightarrow y=\frac{4}{5}x-\frac{14}{5} \Rightarrow \boxed{(t_2): 4x-5y-14=0}$$

f) $\overrightarrow{AB} = \begin{pmatrix} 4+3 \\ 1+2 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$

$\overrightarrow{AD} = \begin{pmatrix} -6+3 \\ 5+2 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$

aire du triangle $ABC = \frac{1}{2}|70 - 12| = 29$

aire du triangle $ADC = \frac{1}{2}|-30 - 28| = 29$

$\Rightarrow \boxed{\text{aire du quadrilatère } ABCD = 58 \text{ u}^2}$

$\overrightarrow{AB} = \begin{pmatrix} 6-1 \\ 2+2 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

$\overrightarrow{AD} = \begin{pmatrix} -3-1 \\ 3+2 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

aire du triangle $ABC = \frac{1}{2}|45 - 4| = \frac{41}{2}$

aire du triangle $ADC = \frac{1}{2}|-36 - 5| = \frac{41}{2}$

$\Rightarrow \boxed{\text{aire du quadrilatère } ABCD = 41 \text{ u}^2}$

Exercice 2.

a) $\int x^{\frac{1}{3}}\, \mathrm{d}x = \frac{3}{4}\, x^{\frac{4}{3}} + c = \boxed{\frac{3}{4}\sqrt[3]{x^4} + c}$

$\boxed{\frac{5}{4}\, x^4 - \frac{7}{3}\, x^3 + 2\, x^2 - 3\, x + c}$

b) $\boxed{\frac{3}{5}\, x^5 + \frac{3}{2}\, x^4 - \frac{1}{3}\, x^3 + x + c}$

$\int x^{\frac{1}{2}}\, \mathrm{d}x = \frac{2}{3}\, x^{\frac{3}{2}} + c = \boxed{\frac{2}{3}\sqrt{x^3} + c}$

c) $\frac{1}{4}\int \underbrace{4}_{u'} \cdot \sin(4x)\, \mathrm{d}x = \boxed{-\frac{1}{4}\, \cos(4\, x) + c}$

$\frac{5}{6}\int \underbrace{6x}_{u'} \cdot (3x^2+1)^4\, \mathrm{d}x = \boxed{\frac{1}{6}\left(3\, x^2 + 1\right)^5 + c}$

d) $\frac{3}{8}\int \underbrace{8x}_{u'} \cdot (4x^2-1)^8\, \mathrm{d}x = \boxed{\frac{1}{24}\left(4\, x^2 - 1\right)^9 + c}$

$\frac{1}{5}\int \underbrace{5}_{u'} \cdot \cos(5x)\, \mathrm{d}x = \boxed{\frac{1}{5}\, \sin(5\, x) + c}$

e) $\frac{1}{2}\int \underbrace{2(x-1)}_{u'} \cdot (x^2-2x)^{-\frac{3}{4}}\, \mathrm{d}x$

$= 2\left(x^2 - 2\, x\right)^{\frac{1}{4}} + c = \boxed{2\sqrt[4]{x^2 - 2x} + c}$

$\int (x^6 - 2x^3 + 1)\, \mathrm{d}x = \boxed{\frac{1}{7}\, x^7 - \frac{1}{2}\, x^4 + x + c}$

f) $\displaystyle\int (x^8 - 2x^4 + 1)\,\mathrm{d}x = \boxed{\frac{1}{9}\,x^9 - \frac{2}{5}\,x^5 + x + c}$

$$\frac{1}{2}\int \underbrace{2(x-3)}_{u'} \cdot (x^2-6x)^{-\frac{4}{5}}\,\mathrm{d}x$$

$$= \frac{5}{2}\left(x^2 - 6\,x\right)^{\frac{1}{5}} + c = \boxed{\frac{5}{2}\sqrt[5]{x^2-6x} + c}$$