Algèbre linéaire I

Exercice 1.

$$A \cdot B = \boxed{\begin{pmatrix} 1 & 7 & -2 \\ 6 & 5 & 7 \end{pmatrix}}$$

Exercice 2.

a) $A^2 = \begin{pmatrix} 3 & -4 & 2 \\ 1 & -1 & -1 \\ 1 & 2 & 0 \end{pmatrix}$ $\quad A^3 = \begin{pmatrix} 5 & 0 & 2 \\ 0 & 3 & 1 \\ 1 & -2 & 4 \end{pmatrix}$ $\quad A^3 - A = \boxed{\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}} = 4I_3$

b) $\frac{1}{4}(A^3 - A) = \frac{1}{4}(A^2 - I_3) \cdot A = I_3 \quad \Leftrightarrow \quad A^{-1} = \frac{1}{4}(A^2 - I_3) = \boxed{\frac{1}{4}\begin{pmatrix} 2 & -4 & 2 \\ 1 & -2 & -1 \\ 1 & 2 & -1 \end{pmatrix}}$

Exercice 3.

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -4 & 1 \\ 3 & -5 & 2 \end{pmatrix} \qquad \det(A) = -3 - 22 + 30 = 5 \qquad A^{-1} = \frac{1}{5}\begin{pmatrix} -3 & -11 & 10 \\ -1 & -7 & 5 \\ 2 & -1 & 0 \end{pmatrix}$$

$$\frac{1}{5}\begin{pmatrix} -3 & -11 & 10 \\ -1 & -7 & 5 \\ 2 & -1 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ 5 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \quad \Rightarrow \quad \mathcal{S} = \boxed{\{(2;0;1)\}}$$

Exercice 4.

$$\left(\begin{array}{cccc|c} 1 & 2 & -5 & 4 & 1 \\ 2 & -3 & 2 & 3 & 18 \\ 1 & 1 & -1 & 1 & 1 \end{array}\right) \xrightarrow[L3 \to L1 - L3]{L2 \to L2 - 2 \cdot L1} \left(\begin{array}{cccc|c} 1 & 2 & -5 & 4 & 1 \\ 0 & -7 & 12 & -5 & 16 \\ 0 & 1 & -4 & 3 & 0 \end{array}\right) \xrightarrow[L2 \to L3]{L3 \to L2 + 7 \cdot L3} \left(\begin{array}{cccc|c} 1 & 2 & -5 & 4 & 1 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & -16 & 16 & 16 \end{array}\right)$$

$$\xrightarrow{L2 \to -\frac{1}{16} \cdot L3} \left(\begin{array}{cccc|c} 1 & 2 & -5 & 4 & 1 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right) \xrightarrow[L2 \to L2 + 4 \cdot L3]{L1 \to L1 + 5 \cdot L3} \left(\begin{array}{cccc|c} 1 & 2 & 0 & -1 & -4 \\ 0 & 1 & 0 & -1 & -4 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right)$$

$$\xrightarrow{L1 \to L1 - 2 \cdot L2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & -1 & -4 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right)$$

$t = k$ (variable libre) $\Rightarrow \quad \mathcal{S} = \boxed{\{(4;-4;-1;0) + k(-1;1;1;1) | k \in \mathbb{R}\}}$