Intégrales I

Exercice 1.

$$f(x) = \sin(x) \quad \Rightarrow \quad f'(x) = \cos(x)$$
$$g'(x) = \sin(x) \quad \Rightarrow \quad gx) = -\cos(x)$$

$$\Rightarrow \quad \int_0^{2\pi} \sin^2(x)\ \mathrm{d}x = -\sin(x)\cos(x)\Big|_0^{2\pi} - \int_0^{2\pi} [-\cos^2(x)]\ \mathrm{d}x = -\sin(x)\cos(x)\Big|_0^{2\pi} + \int_0^{2\pi} \cos^2(x)\ \mathrm{d}x$$

$$\Rightarrow \quad \int_0^{2\pi} \sin^2(x)\ \mathrm{d}x = -\sin(x)\cos(x)\Big|_0^{2\pi} + \int_0^{2\pi} 1\ \mathrm{d}x - \int_0^{2\pi} \sin^2(x)\ \mathrm{d}x$$

$$\Rightarrow \quad \int_0^{2\pi} \sin^2(x)\ \mathrm{d}x = -\sin(x)\cos(x)\Big|_0^{2\pi} + x\Big|_0^{2\pi} - \int_0^{2\pi} \sin^2(x)\ \mathrm{d}x$$

$$\Rightarrow \quad 2\int_0^{2\pi} \sin^2(x)\ \mathrm{d}x = (x - \sin(x)\cos(x))\Big|_0^{2\pi}$$

$$\Rightarrow \quad \int_0^{2\pi} \sin^2(x)\ \mathrm{d}x = \frac{1}{2}\,(x - \sin(x)\cos(x))\Big|_0^{2\pi} = \frac{1}{2}(2\pi - 0 - 0 + 0) = \boxed{\pi}$$

Exercice 2.

a) $$\int_0^1 (x^4 - 2x^3 + x^2 - 4x + 3)\ \mathrm{d}x = \frac{1}{5}\,x^5 - \frac{1}{2}\,x^4 + \frac{1}{3}\,x^3 - 2\,x^2 + 3\,x\Big|_0^1 = \boxed{\frac{31}{30}}$$

b) $$\int_0^3 \frac{2x+1}{\sqrt{1+2x+2x^2}}\ \mathrm{d}x = \frac{1}{2}\int_0^3 \frac{\overbrace{4x+2}^{u'}}{\underbrace{\sqrt{1+2x+2x^2}}_{\sqrt{u}}}\ \mathrm{d}x = \sqrt{2\,x^2 + 2\,x + 1}\Big|_0^3 = \boxed{4}$$

c) $$\int_2^3 \pi x \cdot (3x^2 - 7)^4\ \mathrm{d}x = \frac{\pi}{6}\int_2^3 \underbrace{6x}_{u'} \cdot \underbrace{(3x^2 - 7)}_{u}{}^4\ \mathrm{d}x = \frac{\pi}{6} \cdot \frac{1}{5}\left(3\,x^2 - 7\right)^5\Big|_2^3 = \boxed{\frac{213125}{2} \cdot \pi}$$

d) $x = t^2 + 2 \quad \Rightarrow \quad \mathrm{d}x = 2t \cdot \mathrm{d}t$

$x = 3 \quad \Rightarrow \quad t = 1$ et $x = 6 \quad \Rightarrow \quad t = 2$

$$\int_3^6 \frac{x^2}{\sqrt{x-2}}\ \mathrm{d}x = \int_1^2 \frac{(t^2+2)^2}{t} \cdot 2t\ \mathrm{d}t = \int_1^2 (2\,t^4 + 8\,t^2 + 8)\ \mathrm{d}t = \frac{2}{5}\,t^5 + \frac{8}{3}\,t^3 + 8\,t\Big|_1^2 = \boxed{\frac{586}{15}}$$

e) $\displaystyle\int_1^9 (1+x^2)\sqrt{x}\,\mathrm{d}x = \int_1^9 (\sqrt{x}+x^{\frac{5}{2}})\,\mathrm{d}x = \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{7}x^{\frac{7}{2}}\Big|_1^9 = \frac{4500}{7} - \frac{20}{21} = \boxed{\frac{13480}{21}}$

f) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \underbrace{\cos(x)}_{u'} \cdot \underbrace{\sin^3(x)}_{u^3}\,\mathrm{d}x = \frac{1}{4}\sin^4(x)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{1}{4} - \frac{1}{64} = \boxed{\frac{15}{64}}$