Applications de la dérivée

Exercice 1.

a) $\lim\limits_{x\to 2} f(x) \overset{\text{"}\frac{0}{0}\text{" (B-H)}}{\overbrace{=}} \lim\limits_{x\to 2} \dfrac{2\,x}{3\,x^2-1} = \boxed{\dfrac{4}{11}}$

$\lim\limits_{x\to 3} f(x) \overset{\text{"}\frac{0}{0}\text{" (B-H)}}{\overbrace{=}} \lim\limits_{x\to 3} \dfrac{2\,x}{3\,x^2-7} = \boxed{\dfrac{3}{10}}$

b) $\lim\limits_{x\to 0} f(x) \overset{\text{"}\frac{0}{0}\text{" (B-H)}}{\overbrace{=}} \lim\limits_{x\to 0} \dfrac{-\cos(x)+1}{10\,x}$

$\overset{\text{"}\frac{0}{0}\text{" (B-H)}}{\overbrace{=}} \lim\limits_{x\to 0} \dfrac{\sin(x)}{10} = \boxed{0}$

$\lim\limits_{x\to 0} f(x) \overset{\text{"}\frac{0}{0}\text{" (B-H)}}{\overbrace{=}} \lim\limits_{x\to 0} \dfrac{6\,x}{\sin(x)}$

$\overset{\text{"}\frac{0}{0}\text{" (B-H)}}{\overbrace{=}} \lim\limits_{x\to 0} \dfrac{6}{\cos(x)} = \boxed{6}$

Exercice 2.

$f(x) = \dfrac{1}{x+1} \qquad g(x) = \dfrac{3}{2x+1}$

$2x+1 = 3x+3 \quad \Leftrightarrow \quad x=-2$

$f'(x) = -\dfrac{1}{(x+1)^2} \qquad g'(x) = -\dfrac{6}{(2x+1)^2}$

$m_1 = f'(-2) = -1 \qquad m_2 = g'(-2) = -\dfrac{2}{3}$

$\tan(\alpha) = \left|\dfrac{-2/3+1}{1+2/3}\right| = \dfrac{1}{5} \Rightarrow \boxed{\alpha \cong 11,31°}$

$f(x) = \dfrac{1}{x+1} \qquad g(x) = \dfrac{4}{3x+5}$

$3x+5 = 4x+4 \quad \Leftrightarrow \quad x=1$

$f'(x) = -\dfrac{1}{(x+1)^2} \qquad g'(x) = -\dfrac{12}{(3x+5)^2}$

$m_1 = f'(1) = -\dfrac{1}{4} \qquad m_2 = g'(1) = -\dfrac{3}{16}$

$\tan(\alpha) = \left|\dfrac{-3/16+1/4}{1+3/64}\right| = \dfrac{4}{67}$

$\Rightarrow \boxed{\alpha \cong 3,42°}$

Exercice 3.

$\boxed{ED(f) = \mathbb{R} - \{5\}}$

$\Rightarrow$ zéro : $\boxed{x=3}$

x	$-\infty$		3		5		$+\infty$
$f(x)$		$-$	0	$-$	||	$+$	

$\boxed{ED(f) = \mathbb{R} - \{-1\}}$

$\Rightarrow$ zéro : $\boxed{x=1 \text{ et } x=7}$

x	$-\infty$		-1		1		7		$+\infty$
$f(x)$		$-$	||	$+$	0	$-$	0	$+$	

$$\lim_{\substack{x\to 5\\ <}} f(x) \overbrace{=}^{\text{"}\frac{4}{0_-}\text{"}} -\infty \text{ et } \lim_{\substack{x\to 5\\ >}} f(x) \overbrace{=}^{\text{"}\frac{4}{0_+}\text{"}} +\infty$$

$\Rightarrow$ $\boxed{\text{AV : } x = 5}$

AO $\Rightarrow$ division polynomiale

$$\begin{array}{l} x^2 - 6x + 9 = (x-5)\,(x-1)\; + 4 \\ -\,x^2 + 5x \\ \hline \quad -\,x + 9 \\ \quad\;\; x - 5 \\ \hline \qquad\quad 4 \end{array}$$

$\Rightarrow$ $\boxed{\text{AO : } y = x - 1}$

étude du signe de $\delta(x) = \dfrac{4}{x-5}$

x	$-\infty$		5		$+\infty$
$\delta(x)$		$-$	‖	$+$	

f est dessus l'AO si $x \in]5; +\infty[$
f est dessous l'AO si $x \in]-\infty; 5[$

$$f'(x) = \frac{(2x-6)\cdot(x-5) - (x^2-6x+9)\cdot 1}{(x-5)^2}$$
$$= \frac{x^2 - 10x + 21}{(x-5)^2} = \frac{(x-3)(x-7)}{(x-5)^2}$$

$ED(f') = ED(f)$

x	$-\infty$		3		5		7		$+\infty$
$f'(x)$		$+$	0	$-$	‖	$-$	0	$+$	
$f(x)$		↗	0	↘	‖	↘	8	↗	

$$\lim_{\substack{x\to -1\\ <}} f(x) \overbrace{=}^{\text{"}\frac{16}{0_-}\text{"}} -\infty \text{ et } \lim_{\substack{x\to -1\\ >}} f(x) \overbrace{=}^{\text{"}\frac{16}{0_+}\text{"}} +\infty$$

$\Rightarrow$ $\boxed{\text{AV : } x = -1}$

AO $\Rightarrow$ division polynomiale

$$\begin{array}{l} x^2 - 8x + 7 = (x+1)\,(x-9)\; + 16 \\ -\,x^2 \;\; - x \\ \hline \quad -\,9x + 7 \\ \quad\;\; 9x + 9 \\ \hline \qquad\quad 16 \end{array}$$

$\Rightarrow$ $\boxed{\text{AO : } y = x - 9}$

étude du signe de $\delta(x) = \dfrac{16}{x+1}$

x	$-\infty$		-1		$+\infty$
$\delta(x)$		$-$	‖	$+$	

f est dessus l'AO si $x \in]-1; +\infty[$
f est dessous l'AO si $x \in]-\infty; -1[$

$$f'(x) = \frac{(2x-8)\cdot(x+1) - (x^2-8x+7)}{(x+1)^2}$$
$$= \frac{x^2 + 2x - 15}{(x+1)^2} = \frac{(x+5)(x-3)}{(x+1)^2}$$

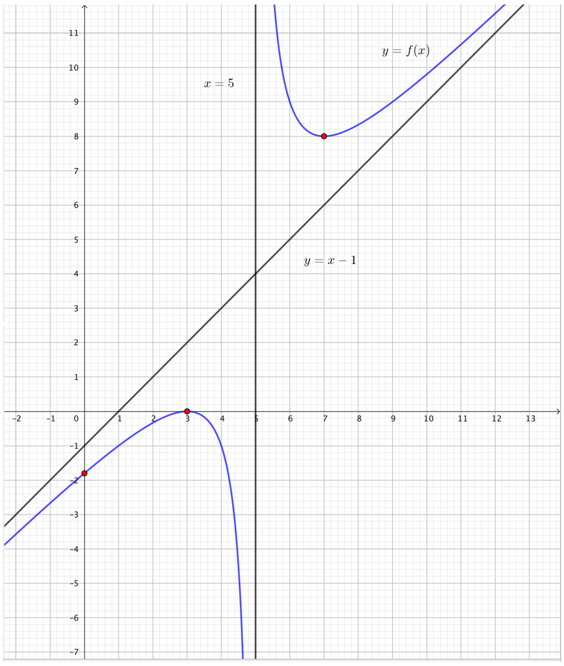
$ED(f') = ED(f)$

x	$-\infty$		-5		-1		3		$+\infty$
$f'(x)$		$+$	0	$-$	‖	$-$	0	$+$	
$f(x)$		↗	-18	↘	‖	↘	-2	↗	

max : $(3;0)$

min : $(7;8)$

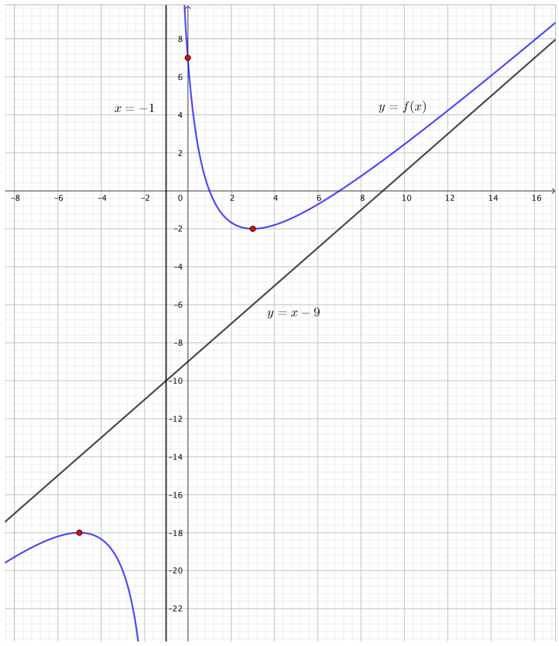
pt. part. : $(0;-\frac{9}{5})$



max : $(-5;-18)$

min : $(3;-2)$

pt. part. : $(0;7)$



Exercice 4.

a) $xy = 45 \quad \Leftrightarrow \quad y = \frac{45}{x}$

$A = (50-3x)(30-y) = 1500-50y-90x+3xy$

$A(x) = 1500 - \frac{2250}{x} - 90x + 135$

$= 1635 - \frac{2250}{x} - 90x$

$= \frac{-90x^2 + 1635x - 2250}{x}$

$$\boxed{= \frac{-15(6x^2 - 109x + 150)}{x}}$$

$xy = 6 \quad \Leftrightarrow \quad y = \frac{6}{x}$

$A = (40-3x)(20-y) = 800-40y-60x+3xy$

$A(x) = 800 - \frac{240}{x} - 60x + 18$

$= 818 - \frac{240}{x} - 60x$

$= \frac{-60x^2 + 818x - 240}{x}$

$$\boxed{= \frac{-2(30x^2 - 409x + 120)}{x}}$$

b) $A'(x) = \dfrac{2250}{x^2} - 90 = \dfrac{2250 - 90^2}{x^2}$

$= \dfrac{90(25 - x^2)}{x^2} = \dfrac{90(5 - x)(5 + x)}{x^2}$

$ED(A) = ED(A') =]0; \frac{50}{3}]$

x	0		5		$\frac{50}{3}$
$A'(x)$		$+$	0	$-$	
$A(x)$		$\nearrow$	735	$\searrow$	

L'aire est maximale si $x = 5$ m

c) $A(5) = 735 \text{ m}^2$

$A'(x) = \dfrac{240}{x^2} - 60 = \dfrac{240 - 60x^2}{x^2}$

$= \dfrac{60(4 - x^2)}{x^2} = \dfrac{60(2 - x)(2 + x)}{x^2}$

$ED(A) = ED(A') =]0; \frac{40}{3}]$

x	0		2		$\frac{40}{3}$
$A'(x)$		$+$	0	$-$	
$A(x)$		$\nearrow$	578	$\searrow$	

L'aire est maximale si $x = 2$ m

$A(2) = 578 \text{ m}^2$