Applications de l'intégrale

Exercice 1.

a) $f(1) = g(1) = 3 = a + 1 \quad \Rightarrow \boxed{a = 2}$

b) $x^2 + 2 = -x^2 + 4 \quad \Rightarrow \quad 2x^2 - 2 = 0$

$\Rightarrow \quad 2(x-1)(x+1) = 0$

$$\int_{-1}^{1} (2x^2 - 2)\,\mathrm{d}x = \left(\frac{2}{3}x^3 - 2x\right)\Big|_{-1}^{1}$$

$$= -\frac{4}{3} - \frac{4}{3} = -\frac{8}{3} \quad \Rightarrow \quad A = \boxed{\frac{8}{3}\ \mathrm{u}^2}$$

$f(2) = g(2) = 5 = a + 4 \quad \Rightarrow \boxed{a = 1}$

$x^2 + 1 = -x^2 + 9 \quad \Rightarrow \quad 2x^2 - 8 = 0$

$\Rightarrow \quad 2(x-2)(x+2) = 0$

$$\int_{-2}^{2} (2x^2 - 8)\,\mathrm{d}x = \left(\frac{2}{3}x^3 - 8x\right)\Big|_{-2}^{2}$$

$$= -\frac{32}{3} - \frac{32}{3} = -\frac{64}{3} \quad \Rightarrow \quad A = \boxed{\frac{64}{3}\ \mathrm{u}^2}$$

Exercice 2.

zéros de f : $x_1 = 0$, $x_2 = -3$ et $x_3 = 3$

$$\frac{1}{2}\int_{-3}^{0} 2x \cdot (x^2 - 9)^4\,\mathrm{d}x = \frac{1}{10}\left(x^2 - 9\right)^5\Big|_{-3}^{0}$$

$$= -\frac{9^5}{10} - 0 = -\frac{59049}{10}$$

$$\frac{1}{2}\int_{0}^{3} 2x \cdot (x^2 - 9)^4\,\mathrm{d}x = \frac{1}{10}\left(x^2 - 9\right)^5\Big|_{0}^{3}$$

$$= 0 - \left(-\frac{9^5}{10}\right) = \frac{59049}{10}$$

$$\Rightarrow \quad \mathcal{A} = \left|-\frac{59049}{10}\right| + \frac{59049}{10} = \boxed{\frac{59049}{5}\ \mathrm{u}^2}$$

zéros de f : $x_1 = 0$, $x_2 = -2$ et $x_3 = 2$

$$\frac{1}{2}\int_{-2}^{0} 2x \cdot (x^2 - 4)^5\,\mathrm{d}x = \frac{1}{12}\left(x^2 - 4\right)^6\Big|_{-2}^{0}$$

$$= \frac{4^6}{12} - 0 = \frac{1024}{3}$$

$$\frac{1}{2}\int_{0}^{2} 2x \cdot (x^2 - 4)^5\,\mathrm{d}x = \frac{1}{12}\left(x^2 - 4\right)^6\Big|_{0}^{2}$$

$$= 0 - \left(\frac{4^6}{12}\right) = -\frac{1024}{3}$$

$$\Rightarrow \quad \mathcal{A} = \frac{1024}{3} + \left|-\frac{1024}{3}\right| = \boxed{\frac{2048}{3}\ \mathrm{u}^2}$$

Exercice 3.

a) $\mathcal{A} = \int_{-2}^{0} \sqrt{1-4x}\,\mathrm{d}x = -\frac{1}{4}\int_{-2}^{0} (-4)(1-4x)^{\frac{1}{2}}\,\mathrm{d}x$

$= -\frac{1}{6}\,(-4\,x+1)^{\frac{3}{2}}\Big|_{-2}^{0} = \boxed{\frac{13}{3}\ \mathrm{u}^2}$

$A = \int_{-4}^{0} \sqrt{1-6x}\,\mathrm{d}x = -\frac{1}{6}\int_{-4}^{0} (-6)(1-6x)^{\frac{1}{2}}\,\mathrm{d}x$

$= -\frac{1}{9}\,(-6\,x+1)^{\frac{3}{2}}\Big|_{-4}^{0} = \boxed{\frac{124}{9}\ \mathrm{u}^2}$

b) $V = \pi\int_{-2}^{0} (1-4x)\,\mathrm{d}x = \pi\left[-2\,x^2+x\right]_{-2}^{0}$

$= \boxed{10\,\pi\ \mathrm{u}^3}$

$V = \pi\int_{-4}^{0} (1-6x)\,\mathrm{d}x = \pi\left[-3\,x^2+x\right]_{-4}^{0}$

$= \boxed{52\,\pi\ \mathrm{u}^3}$

Exercice 4.

a) $-x^2+4x = -x+4 \quad\Rightarrow\quad x^2-5x+4=0$

$\Rightarrow\quad (x-1)(x-4)=0 \quad\Rightarrow\quad x=1, x=4$

$f(1)=g(1)=3$ et $f(4)=g(4)=0$

$\Rightarrow\quad \boxed{A(1;3) \qquad B(4;0)}$

$-x^2+3x = -x+3 \quad\Rightarrow\quad x^2-4x+3=0$

$\Rightarrow\quad (x-1)(x-3)=0 \quad\Rightarrow\quad x=1, x=3$

$f(1)=g(1)=2$ et $f(3)=g(3)=0$

$\Rightarrow\quad \boxed{A(1;2) \qquad B(3;0)}$

b) $\int_{0}^{1} (x^2-5x+4)\,\mathrm{d}x = \frac{1}{3}\,x^3-\frac{5}{2}\,x^2+4\,x\Big|_{0}^{1} = \frac{11}{6}$

$\int_{1}^{4} (x^2-5x+4)\,\mathrm{d}x = \frac{1}{3}\,x^3-\frac{5}{2}\,x^2+4\,x\Big|_{1}^{4} = -\frac{9}{2}$

$\Rightarrow\quad$ aire de $D = \frac{11}{6}+\frac{9}{2} = \boxed{\frac{19}{3}\ \mathrm{u}^2}$

$\int_{0}^{1} (x^2-4x+3)\,\mathrm{d}x = \frac{1}{3}\,x^3-2\,x^2+3\,x\Big|_{0}^{1} = \frac{4}{3}$

$\int_{1}^{3} (x^2-4x+3)\,\mathrm{d}x = \frac{1}{3}\,x^3-2\,x^2+3\,x\Big|_{1}^{3} = -\frac{4}{3}$

$\Rightarrow\quad$ aire de $D = \frac{4}{3}+\frac{4}{3} = \boxed{\frac{8}{3}\ \mathrm{u}^2}$

c)

$$\pi \int_0^1 (-x^2+4x)^2 \,\mathrm{d}x = \pi \int_0^1 (x^4-8\,x^3+16\,x^2) \,\mathrm{d}x$$
$$= \pi \left[\frac{1}{5}\,x^5 - 2\,x^4 + \frac{16}{3}\,x^3\right]_0^1 = \frac{53}{15}\,\pi$$

$$\pi \int_1^4 (-x+4)^2 \,\mathrm{d}x = \pi \int_1^4 (x^2-8\,x+16) \,\mathrm{d}x$$
$$= \pi \left[\frac{1}{3}\,x^3 - 4\,x^2 + 16\,x\right]_1^4 = 9\,\pi$$

$$\Rightarrow \quad V = \frac{53}{15}\,\pi + 9\,\pi = \boxed{\frac{188}{15}\,\pi \text{ u}^3}$$

$$\pi \int_0^1 (-x^2+3x)^2 \,\mathrm{d}x = \pi \int_0^1 (x^4-6\,x^3+9\,x^2) \,\mathrm{d}x$$
$$= \pi \left[\frac{1}{5}\,x^5 - \frac{3}{2}\,x^4 + 3\,x^3\right]_0^1 = \frac{17}{10}\,\pi$$

$$\pi \int_1^3 (-x+3)^2 \,\mathrm{d}x = \pi \int_1^3 (x^2-6\,x+9) \,\mathrm{d}x$$
$$= \pi \left[\frac{1}{3}\,x^3 - 3\,x^2 + 9\,x\right]_1^3 = \frac{8}{3}\,\pi$$

$$\Rightarrow \quad V = \frac{17}{10}\,\pi + \frac{8}{3}\,\pi = \boxed{\frac{131}{30}\,\pi \text{ u}^3}$$