Géométrie analytique II

Exercice 1.

a) $x^2+6x+9+y^2=41+9$

$(x+3)^2+y^2=50$

$\Rightarrow$ $\boxed{K(-3;0) \qquad r=5\sqrt{2}\text{ u}}$

b) $(x+3)^2+(y-6)^2=r_2^2$

$A\in\gamma_2 \quad\Rightarrow\quad 4+4=8=r_2^2$

$\Rightarrow$ $\boxed{(\gamma_2)\ :\ (x+3)^2+(y-6)^2=8}$

c) $T\in\gamma_2\,?: 4+4=8\ \checkmark \quad\Rightarrow\quad T\in\gamma_2$

équ. déd. : $(x+3)(x+3)+(y-6)(y-6)=8$

$\Rightarrow\quad 2(x+3)-2(y-6)=8$

$\Rightarrow$ $\boxed{(t)\ :\ x-y+5=0}$

d) $(g)\ :\ y=mx+h$, $E\in g$ droite

$\Rightarrow\quad 8=3m+h \quad\Rightarrow\quad h=8-3m$

$\Rightarrow\quad y=mx+8-3m$

$\Rightarrow\quad mx-y+8-3m=0$

$\delta(K;g)=\dfrac{|-3m+0+8-3m|}{\sqrt{m^2+1}}=5\sqrt{2}$

$\Rightarrow\quad |-6m+8|=5\sqrt{2}\sqrt{m^2+1}$

$36m^2-96m+64=50m^2+50$

$\Rightarrow\quad 14m^2+96m-14=0$

$\Rightarrow\quad 7m^2+48m-7=0$

$\Rightarrow\quad (7m-1)(m+7)=0$

$\Rightarrow\quad m_1=\dfrac{1}{7} \qquad m_2=-7$

$x^2-6x+9+y^2+4y+4=37+9+4$

$(x-3)^2+(y+2)^2=50$

$\Rightarrow$ $\boxed{K(3;-2) \qquad r=5\sqrt{2}\text{ u}}$

$(x-3)^2+(y-4)^2=r_2^2$

$A\in\gamma_2 \quad\Rightarrow\quad 4+4=8=r_2^2$

$\Rightarrow$ $\boxed{(\gamma_2)\ :\ (x-3)^2+(y-4)^2=8}$

$T\in\gamma_2\,?: 4+4=8\ \checkmark \quad\Rightarrow\quad T\in\gamma_2$

équ. déd. : $(x-3)(x-3)+(y-4)(y-4)=8$

$\Rightarrow\quad -2(x-3)-2(y-4)=8$

$\Rightarrow$ $\boxed{(t)\ :\ x+y-3=0}$

$(g)\ :\ y=mx+h$, $E\in g$ droite

$\Rightarrow\quad 6=-3m+h \quad\Rightarrow\quad h=6+3m$

$\Rightarrow\quad y=mx+6+3m$

$\Rightarrow\quad mx-y+6+3m=0$

$\delta(K;g)=\dfrac{|3m+2+6+3m|}{\sqrt{m^2+1}}=5\sqrt{2}$

$\Rightarrow\quad |6m+8|=5\sqrt{2}\sqrt{m^2+1}$

$36m^2+96m+64=50m^2+50$

$\Rightarrow\quad 14m^2-96m-14=0$

$\Rightarrow\quad 7m^2-48m-7=0$

$\Rightarrow\quad (7m+1)(m-7)=0$

$\Rightarrow\quad m_1=-\dfrac{1}{7} \qquad m_2=7$

$\Rightarrow \quad h_1 = \dfrac{53}{7} \qquad h_2 = 29$

$\Rightarrow \boxed{(g) \ : \ y = \dfrac{1}{7}x + \dfrac{53}{7}}$

$\Rightarrow \boxed{(h) \ : \ y = -7x + 29}$

e) $g \cap \gamma_1 : x = 7y - 53$

$(7y-53)^2 + y^2 + 6(7y-53) - 41 = 0$

$49y^2 - 742y + 2809 + 42y - 318 - 41 = 0$

$\Rightarrow \quad 50y^2 - 700y + 2450 = 0$

$\Rightarrow \quad y^2 - 14y + 49 = 0$

$\Rightarrow \quad (y-7)^2 = 0 \quad \Rightarrow y = 7$

$\Rightarrow x = 49 - 53 = -4 \quad \Rightarrow \boxed{G(-4;7)}$

f) $\overrightarrow{CT} = \begin{pmatrix} -1+3 \\ 4-6 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\overrightarrow{TE} = \begin{pmatrix} 3+1 \\ 8-4 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

$\overrightarrow{CT} \cdot \overrightarrow{TE} = 2 \cdot 4 + (-2) \cdot 4 = 0$

$\Rightarrow \boxed{CT \perp TE}$

g) $\overrightarrow{CT} = \begin{pmatrix} -1+3 \\ 4-6 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} -3+5 \\ 6-4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\det(\overrightarrow{CT}; \overrightarrow{AC}) = 2 \cdot 2 - (-2) \cdot 2 = 8$

$\Rightarrow \boxed{\text{aire } \Delta \ ACT = \dfrac{8}{2} = 4 \text{ u}^2}$

$\Rightarrow \quad h_1 = \dfrac{39}{7} \qquad h_2 = 27$

$\Rightarrow \boxed{(g) \ : \ y = -\dfrac{1}{7}x + \dfrac{39}{7}}$

$\Rightarrow \boxed{(h) \ : \ y = 7x + 27}$

$g \cap \gamma_1 : x = -7y + 39$

$(-7y+39)^2 + y^2 - 6(-7y+39) + 4y - 37 = 0$

$49y^2 - 546y + 1521 + 42y - 234 + 4y - 37 = 0$

$\Rightarrow \quad 50y^2 - 500y + 1250 = 0$

$\Rightarrow \quad y^2 - 10y + 25 = 0$

$\Rightarrow \quad (y-5)^2 = 0 \quad \Rightarrow y = 5$

$\Rightarrow x = -35 + 39 = 4 \quad \Rightarrow \boxed{G(4;5)}$

$\overrightarrow{CT} = \begin{pmatrix} 1-3 \\ 2-4 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$\overrightarrow{TE} = \begin{pmatrix} -3-1 \\ 6-2 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$

$\overrightarrow{CT} \cdot \overrightarrow{TE} = (-2) \cdot 4 + (-2) \cdot (-4) = 0$

$\Rightarrow \boxed{CT \perp TE}$

$\overrightarrow{CT} = \begin{pmatrix} 1-3 \\ 2-4 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 3-5 \\ 4-2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$\det(\overrightarrow{CT}; \overrightarrow{AC}) = (-2) \cdot 2 - (-2) \cdot (-2) = -8$

$\Rightarrow \boxed{\text{aire } \Delta \ ACT = \dfrac{8}{2} = 4 \text{ u}^2}$

Exercice 2.

a) $\displaystyle\int x^{\frac{1}{4}}\,\mathrm{d}x = \frac{4}{5}\,x^{\frac{5}{4}} + c = \boxed{\frac{4}{5}\sqrt[4]{x^5} + c}$

b) $\boxed{-\frac{3}{5}\,x^5 - x^4 + x^3 + 2\,x + c}$

c) $\displaystyle\frac{1}{5}\int \underbrace{5}_{u'}\cdot\sin(5x)\,\mathrm{d}x = \boxed{-\frac{1}{5}\,\cos\left(5\,x\right) + c}$

d) $\displaystyle\frac{4}{9}\int \underbrace{9x^2}_{u'}\cdot(3x^3-2)^8\,\mathrm{d}x = \boxed{\frac{4}{81}\left(3\,x^3 - 2\right)^9 + c}$

e) $\displaystyle\frac{1}{2}\int \underbrace{2(x-2)}_{u'}\cdot(x^2-4x)^{-\frac{3}{4}}\,\mathrm{d}x$

$= 2\left(x^2 - 4\,x\right)^{\frac{1}{4}} + c = \boxed{2\sqrt[4]{x^2-4x} + c}$

f) $\displaystyle\int (x^{12}-2x^6+1)\,\mathrm{d}x = \boxed{\frac{1}{13}\,x^{13} - \frac{2}{7}\,x^7 + x + c}$

$\boxed{-\frac{1}{2}\,x^4 + 3\,x^3 - \frac{5}{2}\,x^2 + 2\,x + c}$

$\displaystyle\int x^{\frac{1}{5}}\,\mathrm{d}x = \frac{5}{6}\,x^{\frac{6}{5}} + c = \boxed{\frac{5}{6}\sqrt[5]{x^6} + c}$

$\displaystyle\frac{5}{6}\int \underbrace{6x^2}_{u'}\cdot(2x^3+1)^4\,\mathrm{d}x = \boxed{\frac{1}{6}\left(2\,x^3 + 1\right)^5 + c}$

$\displaystyle\frac{1}{6}\int \underbrace{6}_{u'}\cdot\cos(6x)\,\mathrm{d}x = \boxed{\frac{1}{6}\,\sin\left(6\,x\right) + c}$

$\displaystyle\int (x^{10}-2x^5+1)\,\mathrm{d}x = \boxed{\frac{1}{11}\,x^{11} - \frac{1}{3}\,x^6 + x + c}$

$\displaystyle\frac{1}{2}\int \underbrace{2(x-4)}_{u'}\cdot(x^2-8x)^{-\frac{4}{5}}\,\mathrm{d}x$

$= \frac{5}{2}\left(x^2 - 8\,x\right)^{\frac{1}{5}} + c = \boxed{\frac{5}{2}\sqrt[5]{x^2-8x} + c}$