

# Géométrie analytique I

## Exercice 1.

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| <p>a) <math>(\gamma_1) : (x+5)^2 + (y-1)^2 = 74 + 25 + 1 = 100</math></p> <p><math>\Rightarrow C_1(-5; 1)</math> et <math>r_1 = 10</math> u</p> <p><math>(\gamma_2) : (x+5)^2 + (y+6)^2 = -52 + 25 + 36 = 9</math></p> <p><math>\Rightarrow C_2(-5; -6)</math> et <math>r_2 = 3</math> u</p> <p>b) <math>\overrightarrow{C_1C_2} = \begin{pmatrix} -5+5 \\ -6-1 \end{pmatrix} = \begin{pmatrix} 0 \\ -7 \end{pmatrix}</math></p> <p><math>\Rightarrow \ \overrightarrow{C_1C_2}\  = \sqrt{0^2 + (-7)^2} = 7 = r_1 - r_2</math></p> <p><math>\Rightarrow \gamma_1</math> et <math>\gamma_2</math> sont tangents intérieurement</p> <p>c) <math>A \in \gamma</math> car <math>25 + 121 - 50 - 22 = 74</math></p> <p>équ. déd. <math>(x+5)(x+5) + (y-1)(y-1) = 100</math></p> <p><math>(t) : 0 + 10(y-1) = 100</math></p> <p><math>\Rightarrow (t) : y = 11</math></p> | <p><math>(\gamma_1) : (x-2)^2 + (y+7)^2 = 11 + 4 + 49 = 64</math></p> <p><math>\Rightarrow C_1(2; -7)</math> et <math>r_1 = 8</math> u</p> <p><math>(\gamma_2) : (x+3)^2 + (y+7)^2 = -49 + 9 + 49 = 9</math></p> <p><math>\Rightarrow C_2(-3; -7)</math> et <math>r_2 = 3</math> u</p> <p><math>\overrightarrow{C_1C_2} = \begin{pmatrix} -3-2 \\ -7+7 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}</math></p> <p><math>\Rightarrow \ \overrightarrow{C_1C_2}\  = \sqrt{(-5)^2 + 0^2} = 5 = r_1 - r_2</math></p> <p><math>\Rightarrow \gamma_1</math> et <math>\gamma_2</math> sont tangents intérieurement</p> <p><math>A \in \gamma</math> car <math>100 + 49 - 40 - 98 = 11</math></p> <p>équ. déd. <math>(x-2)(x-2) + (y+7)(y+7) = 64</math></p> <p><math>(t) : 8(x-2) + 0 = 64</math></p> <p><math>\Rightarrow (t) : x = 10</math></p> |
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## Exercice 2.

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| <p><math>M</math> milieu de <math>AB : M\left(\frac{-7+1}{2}; \frac{3+5}{2}\right)</math></p> <p><math>\Rightarrow M(-3; 4)</math></p> <p><math>\overrightarrow{AB} = \begin{pmatrix} 1+7 \\ 5-3 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 4 \\ 1 \end{pmatrix}</math></p> <p><math>m</math> médiatrice de <math>AB</math></p> <p><math>(m) : 4x + y + c = 0</math></p> <p><math>M \in m \Rightarrow -12 + 4 + c = 0 \Leftrightarrow c = 8</math></p> <p><math>\Rightarrow (m) : 4x + y + 8 = 0</math></p> | <p><math>M</math> milieu de <math>AB : M\left(\frac{-1+8}{2}; \frac{5+4}{2}\right)</math></p> <p><math>\Rightarrow M(3,5; 4,5)</math></p> <p><math>\overrightarrow{AB} = \begin{pmatrix} 8+1 \\ 4-5 \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \end{pmatrix}</math></p> <p><math>m</math> médiatrice de <math>AB</math></p> <p><math>(m) : 9x - y + c = 0</math></p> <p><math>M \in m \Rightarrow 31,5 - 4,5 + c = 0 \Leftrightarrow c = -27</math></p> <p><math>\Rightarrow (m) : 9x - y - 27 = 0</math></p> |
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$$m \cap d : x = 0 \Rightarrow y = -8$$

$$\Rightarrow (\gamma) : x^2 + (y + 8)^2 = r^2$$

$$A \in \gamma \Rightarrow 49 + 121 = 170$$

$$\Rightarrow (\gamma) : x^2 + (y + 8)^2 = 170$$

$$m \cap d : x = 0 \Rightarrow y = -27$$

$$\Rightarrow (\gamma) : x^2 + (y + 27)^2 = r^2$$

$$A \in \gamma \Rightarrow 1 + 484 = 1025$$

$$\Rightarrow (\gamma) : x^2 + (y + 27)^2 = 1025$$

**Exercice 3.**

$$a) (\gamma) : (x + 4)^2 + (y - 3)^2 = r^2$$

$$A \in \gamma : 9 + 16 = 25 = r^2$$

$$\Rightarrow (\gamma) : (x + 4)^2 + (y - 3)^2 = 25$$

$$b) \delta(C; d) = \frac{|-16 + 9 - 43|}{\sqrt{16 + 9}} = \frac{50}{5} = 10 > 5$$

$$\Rightarrow d \text{ est extérieure à } \gamma$$

$$c) m_d = -\frac{4}{3} \Rightarrow m = \frac{3}{4}$$

$$(t) : y - 3 = \frac{3}{4}(x + 4) \pm 5\sqrt{\frac{9}{16} + 1}$$

$$\Rightarrow y - 3 = \frac{3}{4}(x + 4) \pm \frac{25}{4}$$

$$\oplus 4y - 12 = 3x + 12 + 25$$

$$\Rightarrow (t_1) : 3x - 4y + 49 = 0$$

$$\ominus 4y - 12 = 3x + 12 - 25$$

$$\Rightarrow (t_2) : 3x - 4y - 1 = 0$$

$$(\gamma) : (x - 6)^2 + (y + 1)^2 = r^2$$

$$A \in \gamma : 16 + 9 = 25 = r^2$$

$$\Rightarrow (\gamma) : (x - 6)^2 + (y + 1)^2 = 25$$

$$\delta(C; d) = \frac{|18 - 4 - 64|}{\sqrt{9 + 16}} = \frac{50}{5} = 10 > 5$$

$$\Rightarrow d \text{ est extérieure à } \gamma$$

$$m_d = -\frac{3}{4} \Rightarrow m = \frac{4}{3}$$

$$(t) : y + 1 = \frac{4}{3}(x - 6) \pm 5\sqrt{\frac{16}{9} + 1}$$

$$\Rightarrow y + 1 = \frac{4}{3}(x - 6) \pm \frac{25}{3}$$

$$\oplus 3y + 3 = 4x - 24 + 25$$

$$\Rightarrow (t_1) : 4x - 3y - 2 = 0$$

$$\oplus 3y + 3 = 4x - 24 - 25$$

$$\Rightarrow (t_2) : 4x - 3y - 52 = 0$$