Fonctions rationnelles, équations et systèmes

Exercice 1

a) $a(x) = \dfrac{x^2 - x - 20}{9 - x^2} = \dfrac{(x-5)(x+4)}{(3-x)(3+x)}$ $\qquad ED(a) = \mathbb{R} - \{-3; 3\}$

zéros: -4; 5 $\qquad$ pôles: -3; 3

x	$-\infty$		-4		-3		3		5		$+\infty$
$x-5$	$-$		$-$		$-$		$-$		0	$+$	
$x+4$	$-$	0	$+$		$+$		$+$		$+$		
$3-x$	$+$		$+$		$+$	0	$-$		$-$		
$3+x$	$-$		$-$	0	$+$		$+$		$+$		
$a(x)$	$-$	0	$+$	$\|\|$	$-$	$\|\|$	$+$	0	$-$		

b) $b(x) = \dfrac{-3(2x-5)(3x+1)(-4x+2)}{x^3 - 25x} = \dfrac{-3(2x-5)(3x+1)(-4x+2)}{x(x-5)(x+5)}$

$ED(b) = \mathbb{R}^* - \{-5; 5\}$

zéros: $-\frac{1}{3}$; $\frac{1}{2}$; $\frac{5}{2}$ $\qquad$ pôles: -5; 0; 5

x	$-\infty$		-5		$-\frac{1}{3}$		0		$\frac{1}{2}$		$\frac{5}{2}$		5		$+\infty$
$-3(2x-5)$	$+$		$+$		$+$		$+$		$+$	0	$-$		$-$		
$3x+1$	$-$		$-$	0	$+$		$+$		$+$		$+$		$+$		
$-4x+2$	$+$		$+$		$+$		$+$	0	$-$		$-$		$-$		
x	$-$		$-$		$-$	0	$+$		$+$		$+$		$+$		
$x-5$	$-$		$-$		$-$		$-$		$-$		$-$	0	$+$		
$x+5$	$-$	0	$+$		$+$		$+$		$+$		$+$		$+$		
$b(x)$	$+$	$\|\|$	$-$	0	$+$	$\|\|$	$-$	0	$+$	0	$-$	$\|\|$	$+$		

c) $c(x) = \dfrac{x^2+12x+36}{x^2-4x-12} = \dfrac{(x+6)^2}{(x-6)(x+2)}$ $\qquad ED(c) = \mathbb{R} - \{-2; 6\}$

zéro: -6 $\qquad$ pôles: -2; 6

x	$-\infty$		-6		-2		6		$+\infty$
$(x+6)^2$		$+$	0	$+$		$+$		$+$	
$x-6$		$-$		$-$		$-$	0	$+$	
$x+2$		$-$		$-$	0	$+$		$+$	
$c(x)$		$+$	0	$+$	‖	$-$	‖	$+$	

d) $d(x) = \dfrac{x^2+3x+7}{x^3}$ $\qquad ED(d) = \mathbb{R}^*$

zéro: aucun ($\Delta = -19$) $\qquad$ pôle: 0

x	$-\infty$		0		$+\infty$
x^2+3x+7		$+$		$+$	
x^3		$-$	0	$+$	
$d(x)$		$-$	‖	$+$	

Exercice 2

a) $\dfrac{6}{10-7x} = \dfrac{4}{5-3x}$ $\qquad ED = \mathbb{R} - \{\frac{10}{7}; \frac{5}{3}\}$

$\dfrac{6(5-3x)}{(10-7x)(5-3x)} = \dfrac{4(10-7x)}{(10-7x)(5-3x)}$

$30 - 18x = 40 - 28x \qquad \Rightarrow 10x = 10$

$x = 1 \qquad \boxed{\Rightarrow S = \{1\}}$

b) $\dfrac{5x^2+2}{x^2-9} = \dfrac{5x-4}{x-3}$ $\qquad \dfrac{5x^2+2}{(x-3)(x+3)} = \dfrac{5x-4}{x-3}$ $\qquad ED = \mathbb{R} - \{-3; 3\}$

$\dfrac{5x^2+2}{(x-3)(x+3)} = \dfrac{(5x-4)(x+3)}{(x-3)(x+3)}$

$5x^2 + 2 = 5x^2 + 11x - 12 \qquad \Rightarrow 11x = 14$

$x = \frac{14}{11} \qquad \boxed{\Rightarrow S = \{\frac{14}{11}\}}$

c) $\dfrac{x-1}{x^2-x-6}-\dfrac{x+1}{x^2-4}=\dfrac{6x}{x^2-5x+6} \quad \Rightarrow \dfrac{x-1}{(x-3)(x+2)}-\dfrac{x+1}{(x-2)(x+2)}=\dfrac{6x}{(x-3)(x-2)}$

$ED=\mathbb{R}-\{-2;2;3\}$

$\dfrac{(x-1)(x-2)}{(x-3)(x-2)(x+2)}-\dfrac{(x+1)(x-3)}{(x-3)(x-2)(x+2)}=\dfrac{6x(x+2)}{(x-3)(x-2)(x+2)}$

$x^2-3x+2-(x^2-2x-3)=6x^2+12x \qquad x^2-3x+2-x^2+2x+3=6x^2+12x$

$6x^2+13x-5=0 \qquad \Rightarrow (2x+5)(3x-1)=0$

$\Rightarrow S=\{-\frac{5}{2};\frac{1}{3}\}$

d) $\dfrac{1}{x^2-2x}=1-\dfrac{2}{x} \qquad \dfrac{1}{x(x-2)}=1-\dfrac{2}{x} \qquad ED=\mathbb{R}^*-\{2\}$

$\dfrac{1}{x(x-2)}=\dfrac{x^2-2x}{x(x-2)}-\dfrac{2(x-2)}{x(x-2)}$

$1=x^2-2x-(2x-4) \qquad 1=x^2-2x-2x+4$

$x^2-4x+3 \qquad \Rightarrow (x-3)(x-1)=0$

$\Rightarrow S=\{1;3\}$

Exercice 3

a) $\dfrac{1}{x-1}+\dfrac{x}{x^2-1}>0 \qquad \dfrac{1}{x-1}+\dfrac{x}{(x-1)(x+1)}>0 \qquad ED=\mathbb{R}-\{-1;1\}$

$\dfrac{x+1}{(x-1)(x+1)}+\dfrac{x}{(x-1)(x+1)}>0 \qquad \Rightarrow \dfrac{2x+1}{(x-1)(x+1)}>0$

zéro: $-\frac{1}{2}$ pôles: -1; 1

x	$-\infty$		-1		$-\frac{1}{2}$		1		$+\infty$
$2x+1$		$-$		$-$	0	$+$		$+$	
$x-1$		$-$		$-$		$-$	0	$+$	
$x+1$		$-$	0	$+$		$+$		$+$	
		$-$	‖	$+$	0	$-$	‖	$+$	

$\Rightarrow S=]-1;-\frac{1}{2}[\cup]1;+\infty[$

b) $\dfrac{13}{x+4} \leq \dfrac{15}{2x-3}$ $\qquad ED = \mathbb{R} - \{-4; \frac{3}{2}\}$

$$\frac{13(2x-3)}{(x+4)(2x-3)} \leq \frac{15(x+4)}{(x+4)(2x-3)} \qquad \frac{26x-39}{(x+4)(2x-3)} \leq \frac{15x+60}{(x+4)(2x-3)}$$

$$\frac{11x-99}{(x+4)(2x-3)} \leq 0 \qquad \Rightarrow \frac{11(x-9)}{(x+4)(2x-3)} \leq 0$$

zéro: 9 $\qquad$ pôles: -4; $\frac{3}{2}$

x	$-\infty$		-4		$\frac{3}{2}$		9		$+\infty$
$11(x-9)$		$-$		$-$		$-$	0	$+$	
$x+4$		$-$	0	$+$		$+$		$+$	
$2x-3$		$-$		$-$	0	$+$		$+$	
		$-$	‖	$+$	‖	$-$	0	$+$	

$\Rightarrow S =]-\infty; -4[\cup]\frac{3}{2}; 9]$

c) $\dfrac{x+1}{x-1} - \dfrac{x-2}{x+2} > \dfrac{9}{5}$ $\qquad ED = \mathbb{R} - \{-2; 1\}$

$$\frac{5(x+1)(x+2)}{(x+2)(x-1)} - \frac{5(x-2)(x-1)}{(x+2)(x-1)} > \frac{9(x+2)(x-1)}{5(x+2)(x-1)}$$

$$\frac{5x^2+15x+10}{(x+2)(x-1)} - \frac{5x^2-15x+10}{(x+2)(x-1)} > \frac{9x^2+9x-18}{5(x+2)(x-1)}$$

$$\frac{5x^2+15x+10-5x^2+15x-10}{(x+2)(x-1)} > \frac{9x^2+9x-18}{5(x+2)(x-1)}$$

$$\frac{9x^2-21x-18}{5(x+2)(x-1)} < 0$$

$$\frac{3(3x^2-7x-6)}{5(x+2)(x-1)} < 0$$

$$\frac{3(3x+2)(x-3)}{5(x+2)(x-1)} < 0$$

zéros: $-\frac{2}{3}$; 3 $\qquad$ pôles: -2; 1

x	$-\infty$		-2		$-\frac{2}{3}$		1		3		$+\infty$
$3(3x+2)$	$-$		$-$	0	$+$		$+$		$+$		
$x-3$	$-$		$-$		$-$		$-$	0	$+$		
$5(x+2)$	$-$	0	$+$		$+$		$+$		$+$		
$x-1$	$-$		$-$		$-$	0	$+$		$+$		
	$+$	‖	$-$	0	$+$	‖	$-$	0	$+$		

$\Rightarrow S =]-2; -\frac{2}{3}[\cup]1; 3[$

d) $\dfrac{x^2-3x+2}{x^2-7x+12} > 1$ $\qquad \dfrac{x^2-3x+2}{(x-3)(x-4)} > 1$ $\qquad ED = \mathbb{R} - \{3; 4\}$

$\dfrac{x^2-3x+2}{(x-3)(x-4)} > \dfrac{x^2-7x+12}{(x-3)(x-4)}$ $\qquad \dfrac{4x-10}{(x-3)(x-4)} > 0$ $\qquad \dfrac{2(2x-5)}{(x-3)(x-4)} > 0$

zéro: $\frac{5}{2}$ $\qquad$ pôles: 3; 4

x	$-\infty$		$\frac{5}{2}$		3		4		$+\infty$
$2(2x-5)$	$-$		0	$+$		$+$		$+$	
$x-3$	$-$			$-$	0	$+$		$+$	
$x-4$	$-$			$-$		$-$	0	$+$	
	$-$		0	$+$	‖	$-$	‖	$+$	

$\Rightarrow S =]\frac{5}{2}; 3[\cup]4; +\infty[$

Exercice 4

a) $\sqrt{2x-3} = x-3$ $\qquad ED = [\frac{3}{2}; +\infty[$

$x - 3 \geq 0 \qquad \Rightarrow x \geq 3 \qquad \Rightarrow S \subset [3; +\infty[$

$2x - 3 = x^2 - 6x + 9 \qquad x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0 \qquad \Rightarrow S = \{6\}$

Vérifications: $x = 2 \quad \Rightarrow \sqrt{4-3} = 1 \neq 2 - 3 = 1$

$x = 6 \quad \Rightarrow \sqrt{12-3} = 3 = 6 - 3$

b) $x-\sqrt{x^2+1}=3 \qquad ED=\mathbb{R}$

$x-3=\sqrt{x^2+1}$

$x-3\geq 0 \qquad \Rightarrow x\geq 3 \qquad \Rightarrow S\subset [3;+\infty[$

$x^2-6x+9=x^2+1 \qquad 6x=8$

$x=\frac{4}{3} \qquad \boxed{\Rightarrow S=\emptyset}$

Vérifications: $x=\frac{4}{3} \qquad \Rightarrow \frac{4}{3}-\sqrt{\frac{25}{9}}=\frac{4}{3}-\frac{5}{3}=-\frac{1}{3}\neq 3$

c) $\sqrt{x}=\sqrt{x-1}+2 \qquad ED=[1;+\infty[$

$x=x-1+4\sqrt{x-1}+4 \qquad -3=4\sqrt{x-1}$

$\boxed{\Rightarrow S=\emptyset}$

d) $\sqrt{x+4}-2=\sqrt{x-2} \qquad ED=[2;+\infty[$

$\sqrt{x+4}=\sqrt{x-2}+2$

$x+4=x-2+4\sqrt{x-2}+4 \qquad 2=4\sqrt{x-2} \qquad 1=2\sqrt{x-2}$

$1=4(x-2) \qquad 1=4x-8 \qquad 4x=9$

$x=\frac{9}{4} \qquad \boxed{\Rightarrow S=\{\frac{9}{4}\}}$

Vérifications: $x=\frac{9}{4} \qquad \Rightarrow \sqrt{\frac{25}{4}}-2=\frac{5}{2}-2=\frac{1}{2}=\sqrt{\frac{1}{4}}$

Exercice 5

a) $\begin{cases} x+y+z=9 \\ x+2y+3z=14 \\ 3x+y+z=21 \end{cases}$

$$\begin{array}{r} x+y+z=9 \\ -x-2y-3z=-14 \\ \hline -y-2z=-5 \end{array} \qquad \begin{array}{r} 3x+3y+3z=27 \\ -3x-y-z=-21 \\ \hline 2y+2z=6 \end{array}$$

$$\begin{array}{r} -y-2z=-5 \\ 2y+2z=6 \\ \hline y=1 \end{array}$$

$\Rightarrow z=2 \qquad \Rightarrow x=6$

$\boxed{\Rightarrow S=\{(6;1;2)\}}$

b) $\begin{cases} 2x - y + z = 16 \\ 3x + 2y - z = 5 \\ 9x - 2y + 2z = 40 \end{cases}$

$$\begin{array}{r} 2x - y + z = 16 \\ 3x + 2y - z = 5 \\ \hline 5x + y = 21 \end{array} \qquad \begin{array}{r} 6x + 4y - 2z = 10 \\ 9x - 2y + 2z = 40 \\ \hline 15x + 2y = 50 \end{array}$$

$$\begin{array}{r} 10x + 2y = 42 \\ -15x - 2y = -50 \\ \hline -5x = -8 \end{array}$$

$\Rightarrow x = \frac{8}{5}$ $\Rightarrow y = 13$ $\Rightarrow z = \frac{129}{5}$

$\Rightarrow S = \{(\frac{8}{5}; 13; \frac{129}{5})\}$

Exercice 6

x: somme du premier capital (en CHF)

y: somme du deuxième capital (en CHF)

z: somme du troisième capital (en CHF)

$$\begin{cases} x + y + z = 97'000 \\ \frac{4x}{100} + \frac{5y}{100} + \frac{6z}{100} = 5'000 \\ \frac{5x}{100} + \frac{6y}{100} + \frac{6z}{100} = 5'570 \end{cases} \qquad \begin{cases} x + y + z = 97'000 \\ 4x + 5y + 6z = 500'000 \\ 5x + 6y + 6z = 557'000 \end{cases}$$

$$\begin{array}{r} 6x + 6y + 6z = 582'000 \\ -4x - 5y - 6z = 500'00 \\ \hline 2x + y = 82'000 \end{array} \qquad \begin{array}{r} -4x - 5y - 6z = 500'00 \\ 5x + 6y + 6z = 557'000 \\ \hline x + y = 57'000 \end{array}$$

$$\begin{array}{r} 2x + y = 82'000 \\ -x - y = -57'000 \\ \hline x = 25'000 \end{array}$$

$\Rightarrow y = 32'000$ $\Rightarrow z = 40'000$

les trois capitaux valent CHF 25'000, CHF 32'000 et CHF 40'000

Exercice 7

x: le plus petit nombre relatif $\quad$ y: le nombre relatif "moyen"

z: le plus grand nombre relatif

$$\begin{cases} x+y+z=8 \\ z=4y \\ y+z=40 \end{cases} \qquad z=4y \qquad \Rightarrow 5y=40 \qquad \Rightarrow y=8$$

$\Rightarrow z=32 \qquad \Rightarrow x=-32$

les trois nombres sont -32, 8 et 32

Exercice 8

u: chiffre des unités du premier nombre

d: chiffre des dizaines du premier nombre

c: chiffre des centaine du premier nombre

$$\begin{cases} 100c+10d+u=36(c+d+u) \\ 100d+10c+u=26(c+d+u) \\ 100c+10d+u+100d+10c+u=1'116 \end{cases} \qquad \begin{cases} 64c-26d-35u=0 \\ -16c+74d-25u=0 \\ 110c+110d+2u=1'116 \end{cases}$$

$$\begin{array}{r} 64c-26d-35u=0 \\ -64c+296d-100u=0 \\ \hline 270d-135u=0 \\ 2d-u=0 \end{array} \qquad \begin{array}{r} -880c+4'070d-1'375u=0 \\ 880c+880d+16u=8'928 \\ \hline 4'950d-1'359u=8'928 \\ 550d-151u=992 \end{array}$$

$$\begin{array}{r} 302d-151u=0 \\ -550d+151u=-992 \\ \hline -248d=-992 \end{array}$$

$\Rightarrow d=4 \qquad \Rightarrow u=8 \qquad \Rightarrow c=6$

les deux nombres sont 648 et 468

Exercice 9

n: nombre d'objets achetés

p: prix d'un objet (en CHF)

$$\begin{cases} np=672 \\ (n+3)(p-4)=672 \end{cases} \qquad \begin{cases} np=672 \\ np-4n+3p-12=672 \end{cases}$$

$np=672 \qquad \Rightarrow -4n+3p-12=0 \qquad \Rightarrow n=\frac{3p}{4}-3$

$(\frac{3p}{4}-3)p=672 \qquad \Rightarrow \frac{3p^2}{4}-3p=672$

$3p^2-12p-2688=0 \qquad p^2-4p-896=0 \qquad \Rightarrow (p-32)(p+28)=0$

$\Rightarrow p=32$ (solution -28 à éliminer) $\qquad \Rightarrow n=21$

il a acheté 21 tableaux à CHF 32.-

Exercice 10

a) $\begin{cases} x^2+y^2+xy=7 \\ x-y+5=0 \end{cases} \Rightarrow y=x+5$

$x^2+(x+5)^2+x(x+5)=7 \qquad x^2+x^2+10x+25+x^2+5x=7$

$3x^2+15x+18=0 \qquad 3(x+2)(x+3)=0$

$x=-2 \qquad \Rightarrow y=3$

$x=-3 \qquad \Rightarrow y=2$

$\boxed{\Rightarrow S=\{(-2;3);(-3;2)\}}$

b) $\begin{cases} 2x-y=7 \\ x^2-y^2-2x-y=3 \end{cases} \Rightarrow y=2x-7$

$x^2-(2x-7)^2-2x-(2x-7)=3 \qquad x^2-4x^2+28x-49-2x-2x+7=3$

$3x^2-24x+45=0 \qquad 3(x-3)(x-5)=0$

$x=3 \qquad \Rightarrow y=-1$

$x=5 \qquad \Rightarrow y=3$

$\boxed{\Rightarrow S=\{(3;-1);(5;3)\}}$

c) $\begin{cases} x^2-y^2=96 \\ x+y=16 \end{cases} \Rightarrow y=16-x$

$x^2-(16-x)^2=96 \qquad x^2-256+32x-x^2=96$

$32x=352 \qquad x=11 \qquad \Rightarrow y=5$

$\boxed{\Rightarrow S=\{(11;5)\}}$

d) $\begin{cases} x^2-y^2+2xy+3x+y-6=0 \\ 5x-4y+3=0 \end{cases} \Rightarrow y=\frac{5x+3}{4}$

$x^2-(\frac{5x+3}{4})^2+2x(\frac{5x+3}{4})+3x+\frac{5x+3}{4}-6=0 \qquad x^2-\frac{25x^2+30x+9}{16}+\frac{10x^2+6x}{4}+3x+\frac{5x+3}{4}-6=0$

$16x^2-25x^2-30x-9+40x^2+24x+48x+20x+12-96=0$

$31x^2+62x-93=0 \qquad 31(x+3)(x-1)=0$

$x=-3 \qquad \Rightarrow y=-3$

$x=1 \qquad \Rightarrow y=2$

$\boxed{\Rightarrow S=\{(-3;-3);(1;2)\}}$